TECHNISCHE UNIVERSITÄT DRESDEN

Institute for Statics und Dynamics of Structures

Fuzzy Structural Design

Bernd Möller

Overview

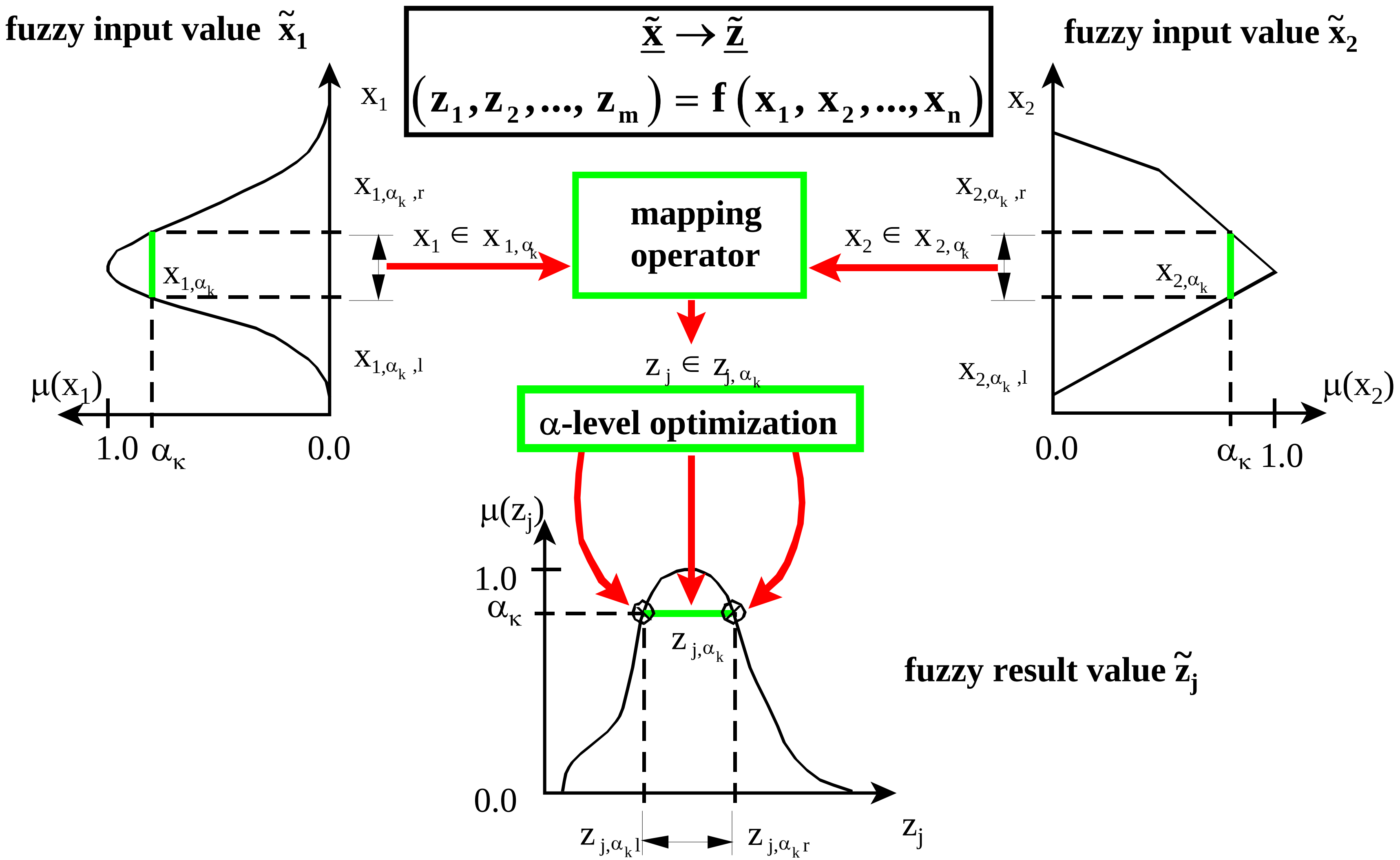
1 Conceptual idea

2 Cluster methods

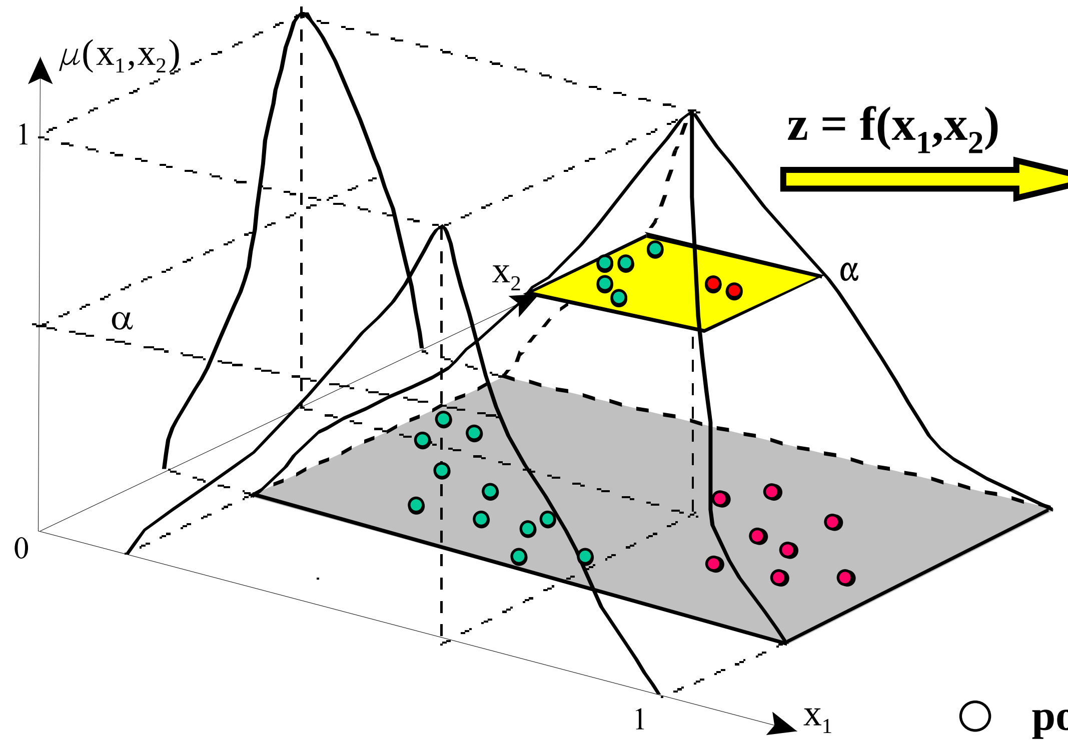
3 Fuzzy cluster design

4 Applications

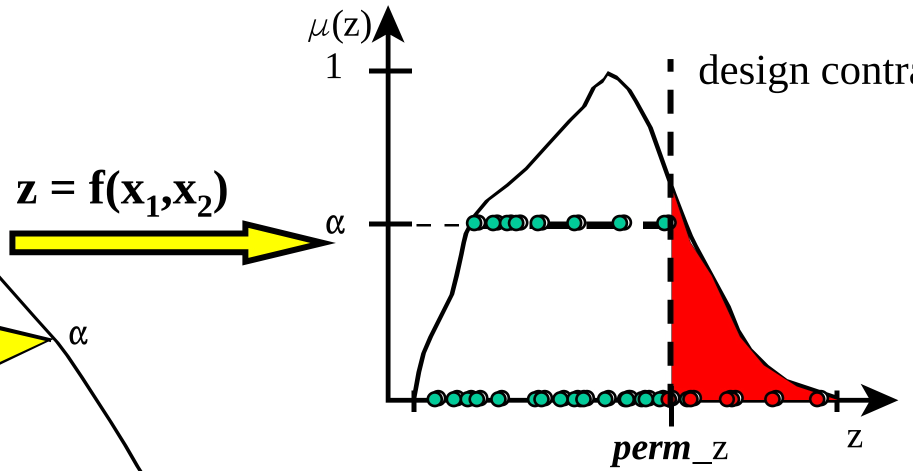
Conceptual idea (1)



Conceptual idea (2)



fuzzy input values $\tilde{x}_1$ and $\tilde{x}_2$



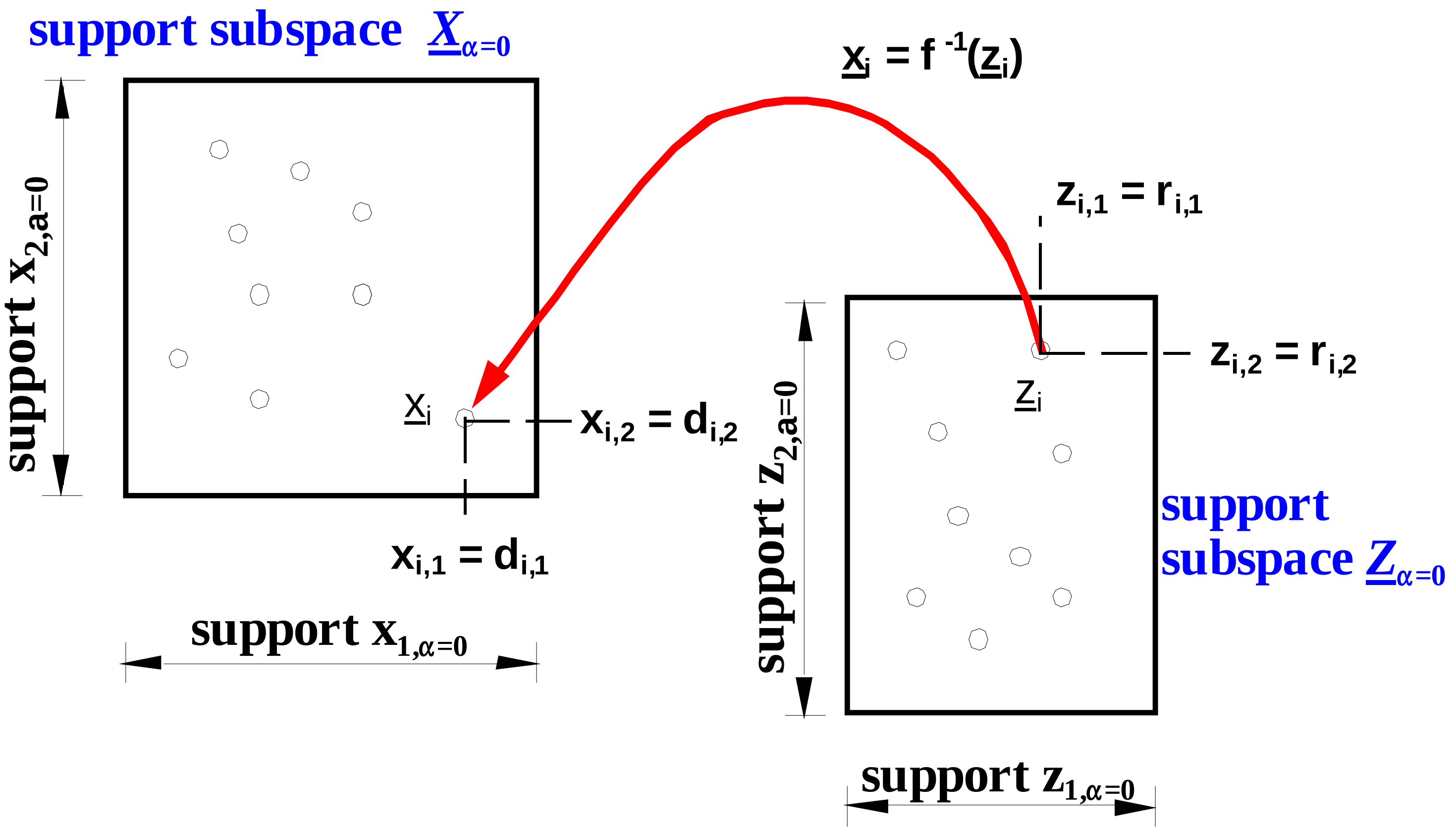
fuzzy result value $\tilde{z}$

$$\underline{z}_i = f\left(\underline{x}_i\right) \leftrightarrow \underline{x}_i = f^{-1}\left(\underline{z}_i\right)$$

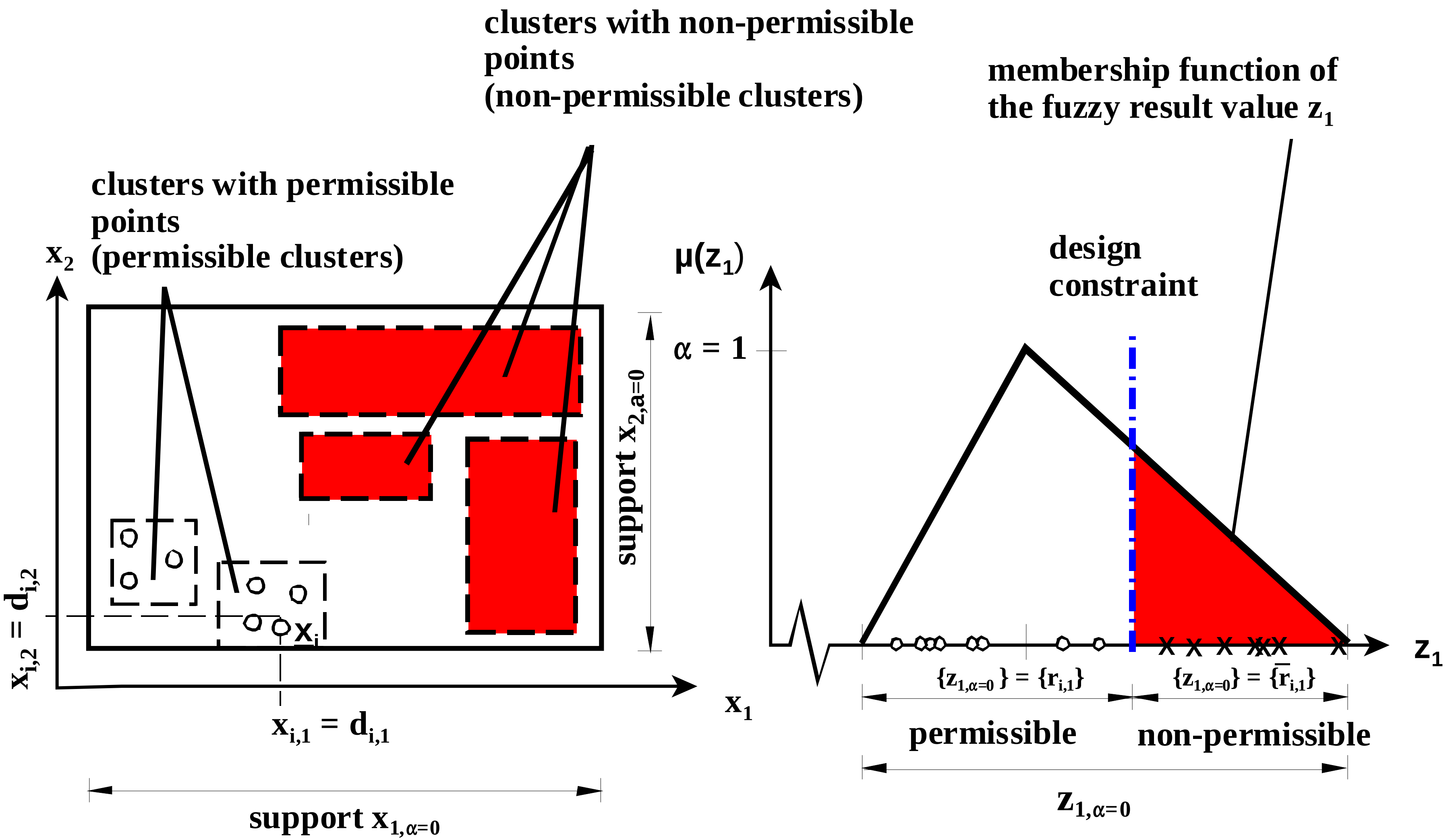
- ○ **points from α- level optimization**
- ● **permissible points**
- ● **non-permissible points**

Conceptual idea (3)

$$\underline{z}_i = f\left(\underline{x}_i\right) \leftrightarrow \underline{x}_i = f^{-1}\left(\underline{z}_i\right)$$



Conceptual idea (4)



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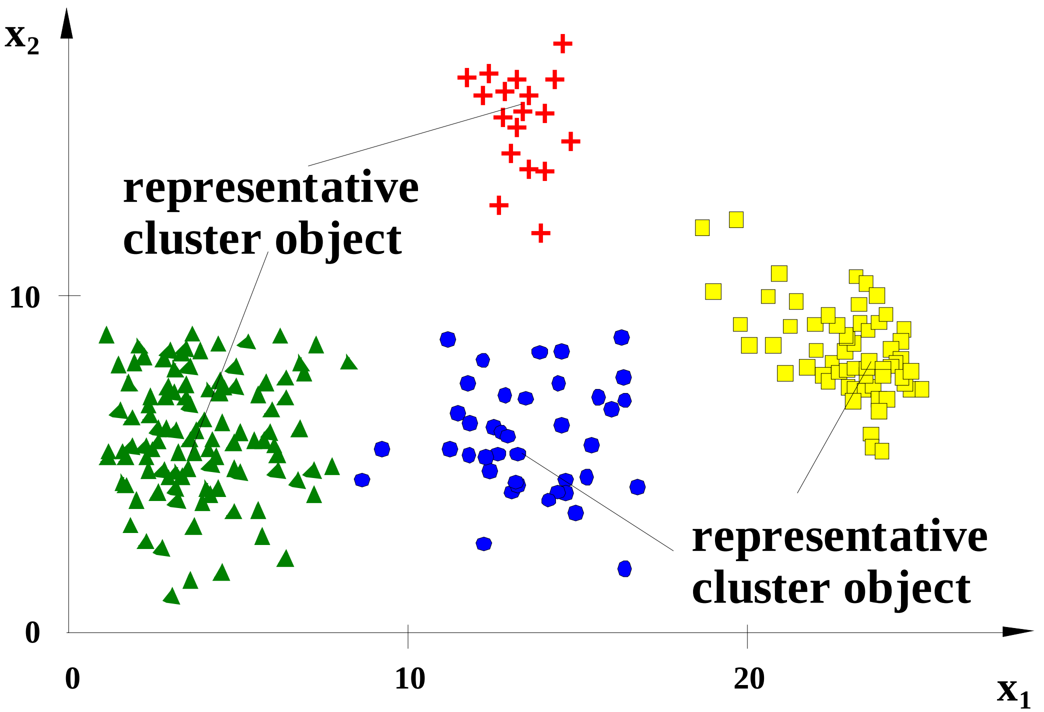
Application of cluster methods (1)

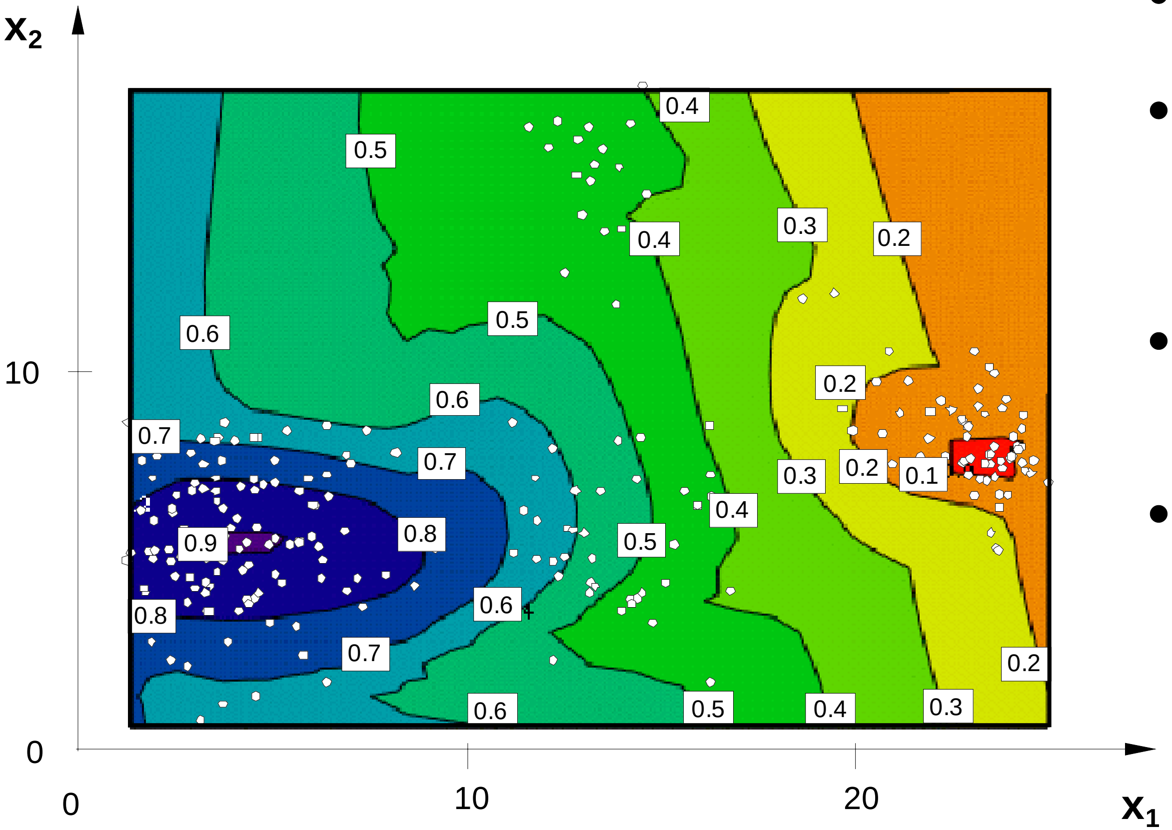
k-medoid cluster method



- k predefined clusters
- selecting of k representative objects and clustering the remaining objects
- improving the set of representative objects and hence clustering
- assessing the quality of clustering by numerical criterions

Application of cluster methods (2)

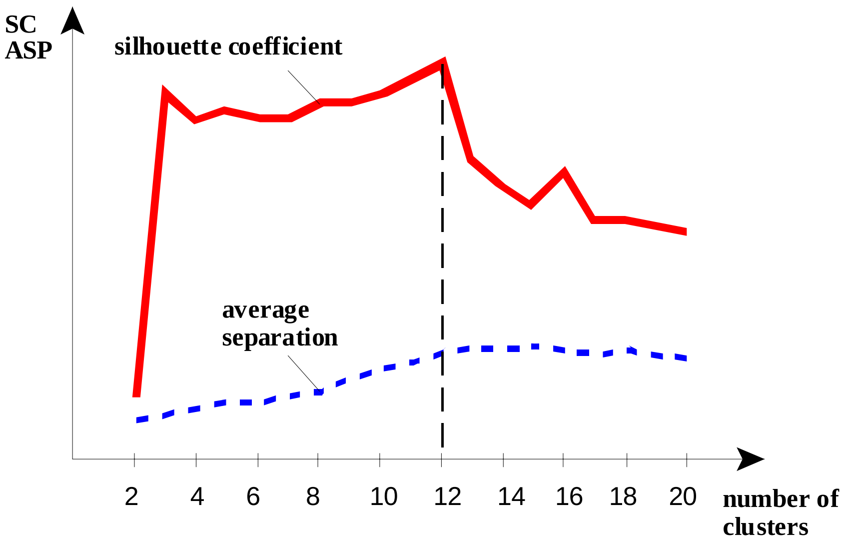
Fuzzy cluster method



- k predefine cluster
- initializing membership values
- iterative improving
  the membership values
  of the objects
- non-linear optimisation
  problem with constraints
- assessing the quality of
  clustering by numerical
  criterions

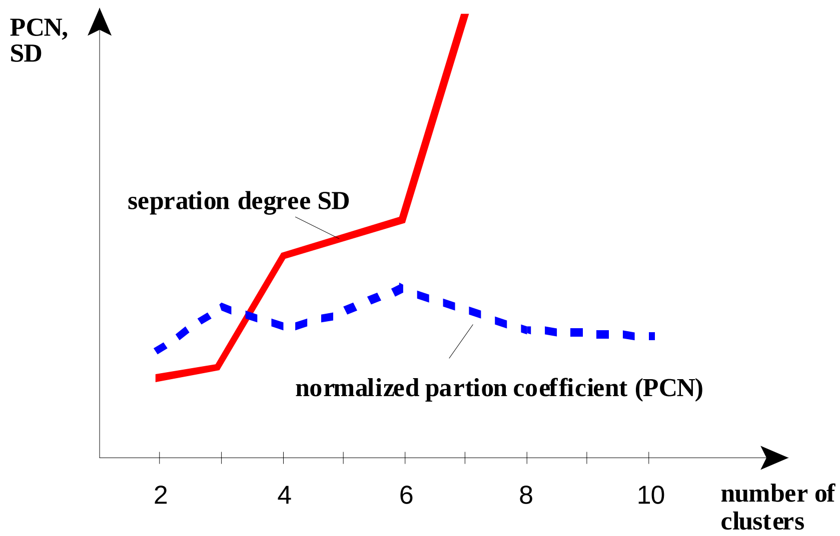
Numerical criterions to assess the quality of clustering

k-medoid method



$$SC = \frac{1}{k}\sum_{v=1}^{k}\frac{1}{m_v}\sum_{i=1}^{m_v}\frac{a_i - b_i}{\max[a_i, b_i]}$$

fuzzy cluster method



$$PCN = 1 - \frac{k}{1-k}\left(1 - \frac{1}{n}\sum_{v=1}^{k}\sum_{i=1}^{n}\mu_{iv}^{2}\right)$$

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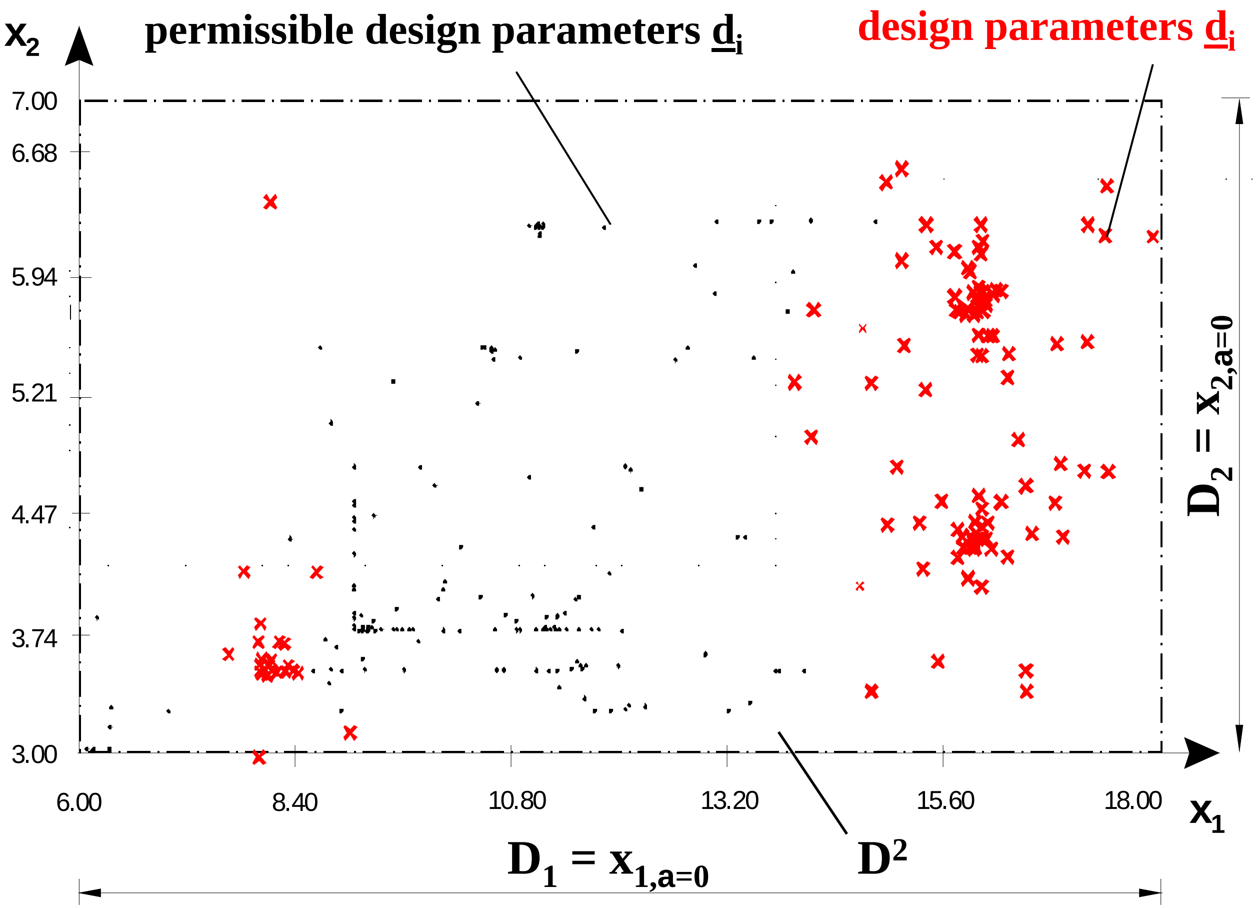
Composition of fuzzy cluster design (1)

Space of design parameters $\underline{D}^n$



$$D_1 = x_{1,\alpha=0}, \ldots, D_u = x_{n,\alpha=0}$$

$$\underline{D}^n = D_1 \times D_2 \ldots D_n$$

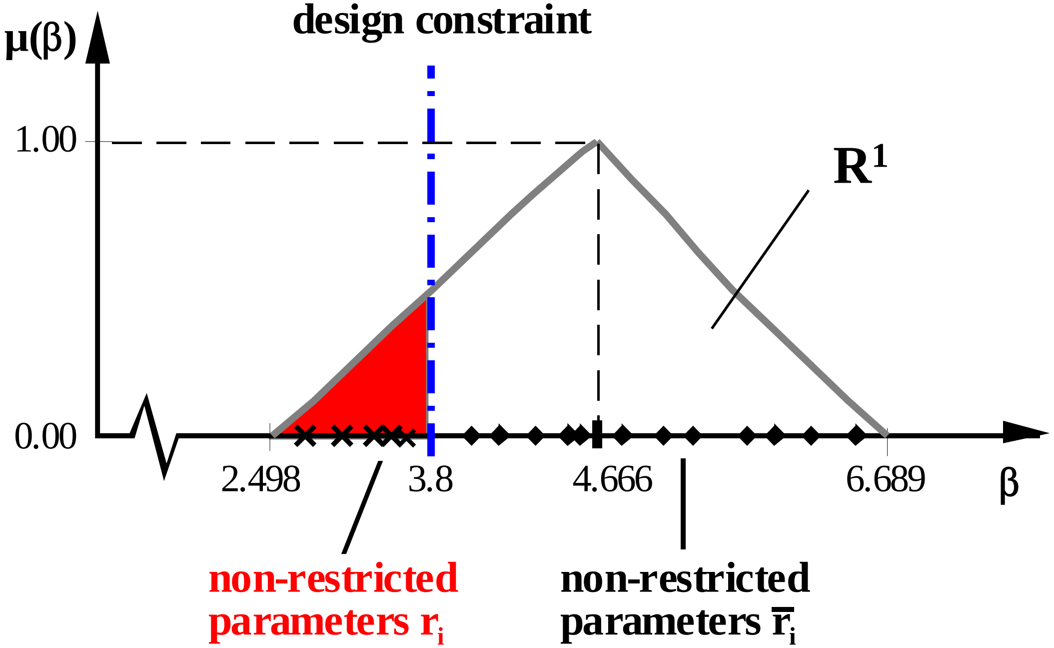
$$M_d = \{\underline{d}_1, \ldots, \underline{d}_i, \ldots, \underline{d}_{nd}\}$$

$$\bar{M}_d = \{\underline{\bar{d}}_1, \ldots, \underline{\bar{d}}_i, \ldots, \underline{\bar{d}}_{\bar{n}d}\}$$

$$M_x = \{M_d, \bar{M}_d\}$$

Composition of fuzzy cluster design (2)

Space of the restricted parameters $\underline{R}^m$



$$R_1 = z_{1,\alpha=0}, \ldots, R_m = z_{m,\alpha=0}$$

$$\underline{R}^m = R_1 \times R_2 \ldots R_m$$

$$M_r = \{\underline{r}_1, \ldots, \underline{r}_i, \ldots, \underline{r}_{\bar{m}r}\}$$

$$\bar{M}_r = \{\underline{\bar{r}}_1, \ldots, \underline{\bar{r}}_i, \ldots, \underline{\bar{r}}_{\bar{m}r}\}$$

$$M_z = \{M_r, \bar{M}_r\}$$

Composition of fuzzy cluster design (3)

Algorithmic procedure

Step I : Initialization of $\underline{D}^n$ and $\underline{R}^m$

Step II: Evaluation of the points in $\underline{R}^m$ by the design constraints

Step III: Determining the permissible points $\underline{d}_i$ and non-permissible points $\bar{\underline{d}}_i$ in $\underline{D}^n \longrightarrow M_d$ and $\bar{M}_d$

Step IV: Clustering the sets M_d and $\bar{M}_d$; result: k_1 permissible cluster

Step V: Constructing of the modified sets $D_1^{[v]}, \ldots, D_n^{[v]}$ from the permissible cluster; result alternative structural design variants

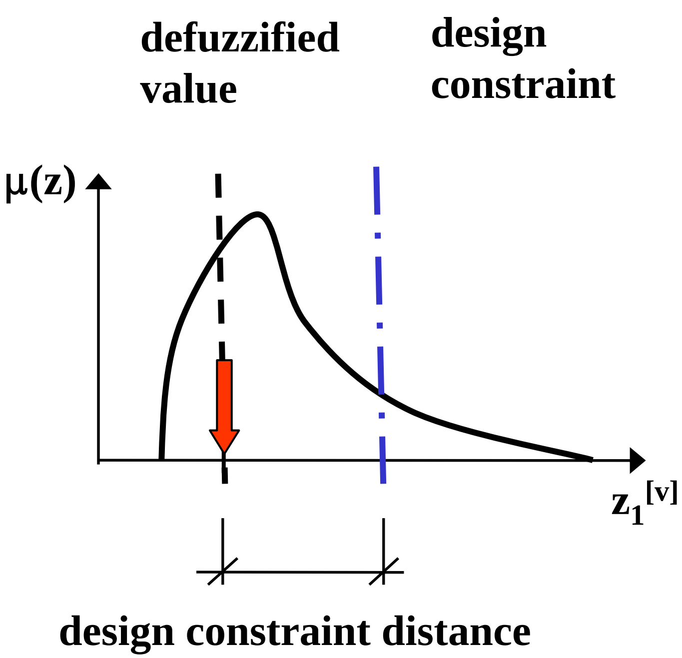
Step VI: Verification of the design variants by α-level optimization $\longrightarrow R_1^{[v]}, \ldots, R_m^{[v]}$

Step VII: Testing, whether all points of the $R_1^{[v]}, \ldots, R_m^{[v]}$ fulfil the constraints

Composition of fuzzy cluster design (4)

Assessment of the alternative permissible design variants by criterions

Criterion I: constraint distance
(measure for distance)



Criterion II: robustness
(measure for sensitivity
of the result variables)

$$B_v = \sum_{j=1}^{m} \sum_{h=1}^{n} \frac{H_n\left(\tilde{z}_j^{[v]}\right)}{H_n\left(\tilde{x}_x^{[v]}\right)}$$

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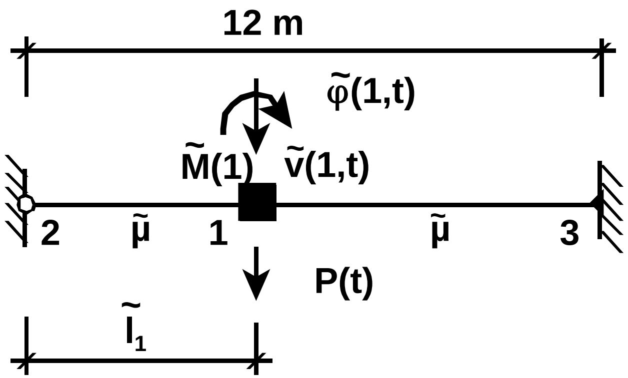
Example 1: Steel girder (1)

Design for time dependent load



dynamic load

$P(t) = P_1 \cdot \cos(\Omega_1 \cdot t) + P_2 \cdot \cos(\Omega_2 \cdot t)$
$P_1 = P_2 = 10$ kN
$\Omega_1 = 44\ s^{-1}$ $\quad \Omega_2 = 66\ s^{-1}$

steel girder

Young´s modulus $E = 2.1 \cdot 10^8$ kN/m²
moment of inertia $I = 1.5 \cdot 10^{-3}\ m^4$

rotational masses are neglected

design parameters:

nodal mass $\quad \tilde{\mathbf{M}}\,(1) = \left\langle 2, \frac{10}{3}, 6 \right\rangle t$

distributed mass $\quad \tilde{\mu} = \left\langle \frac{1}{3}, \frac{5}{9}, 1 \right\rangle \frac{t}{m}$

global mass (full interaction)

$$\tilde{\mathbf{M}} = \tilde{\mathbf{M}}\,(1) + \tilde{\mu} \cdot 12\,m = \langle 6, 10, 18 \rangle\, t$$

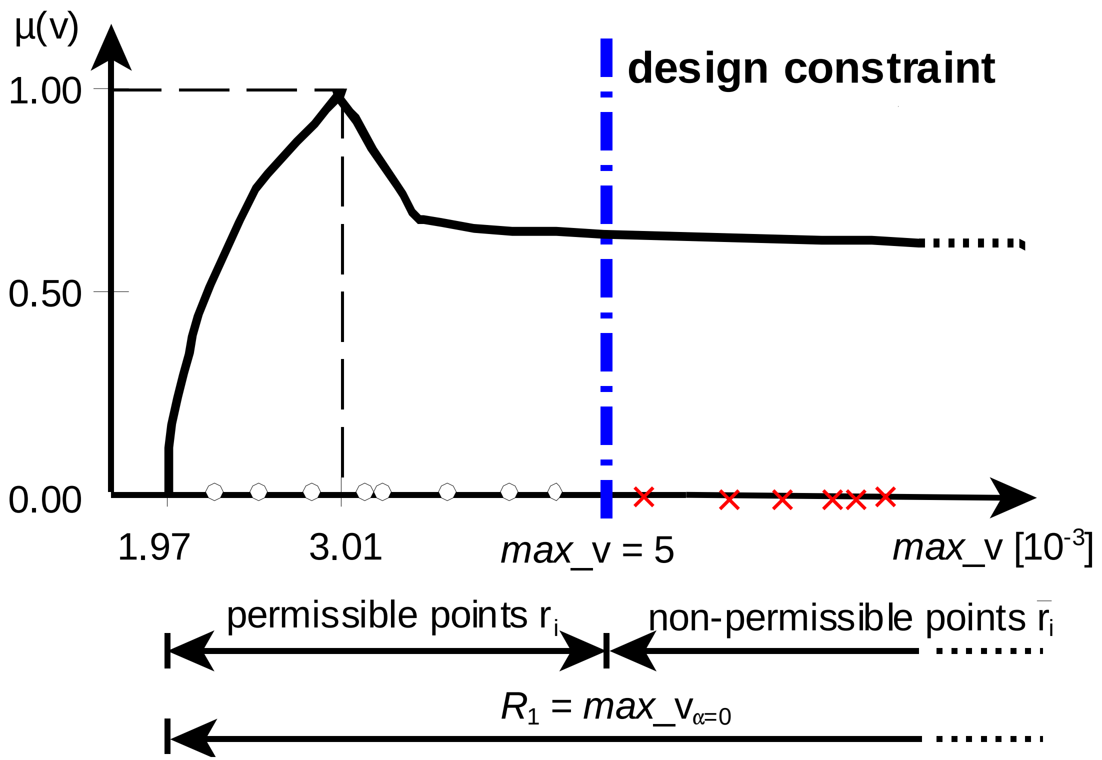
distance

$\tilde{\mathbf{l}}_1$

Example 1: Steel girder (2)

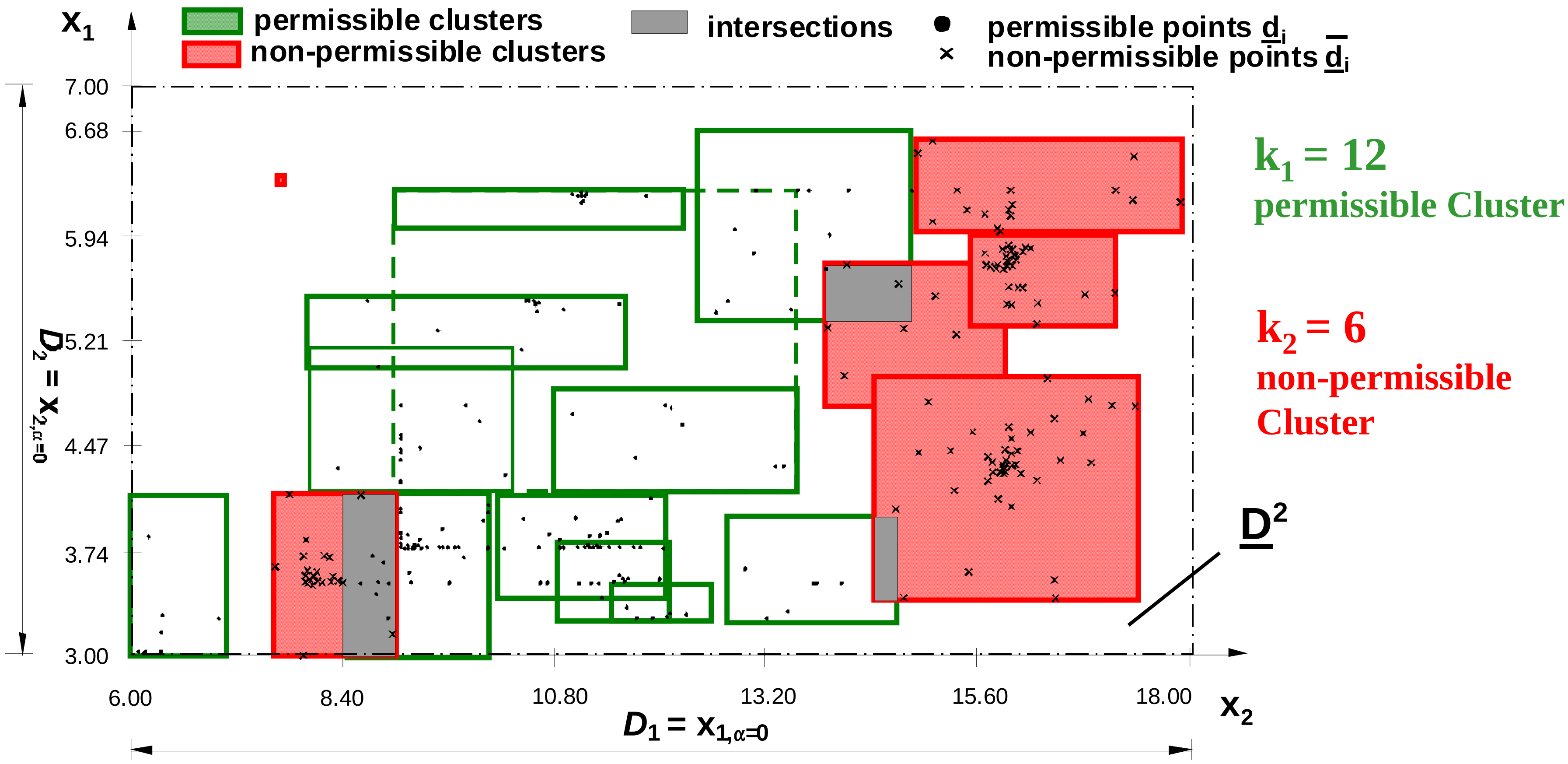
Restricted parameter: displacement norm

$$\tilde{v}(t) = \sqrt{\frac{\tilde{v}(1,t)^2}{1^2} + \frac{\tilde{\varphi}(1,t)^2}{r^2}}$$

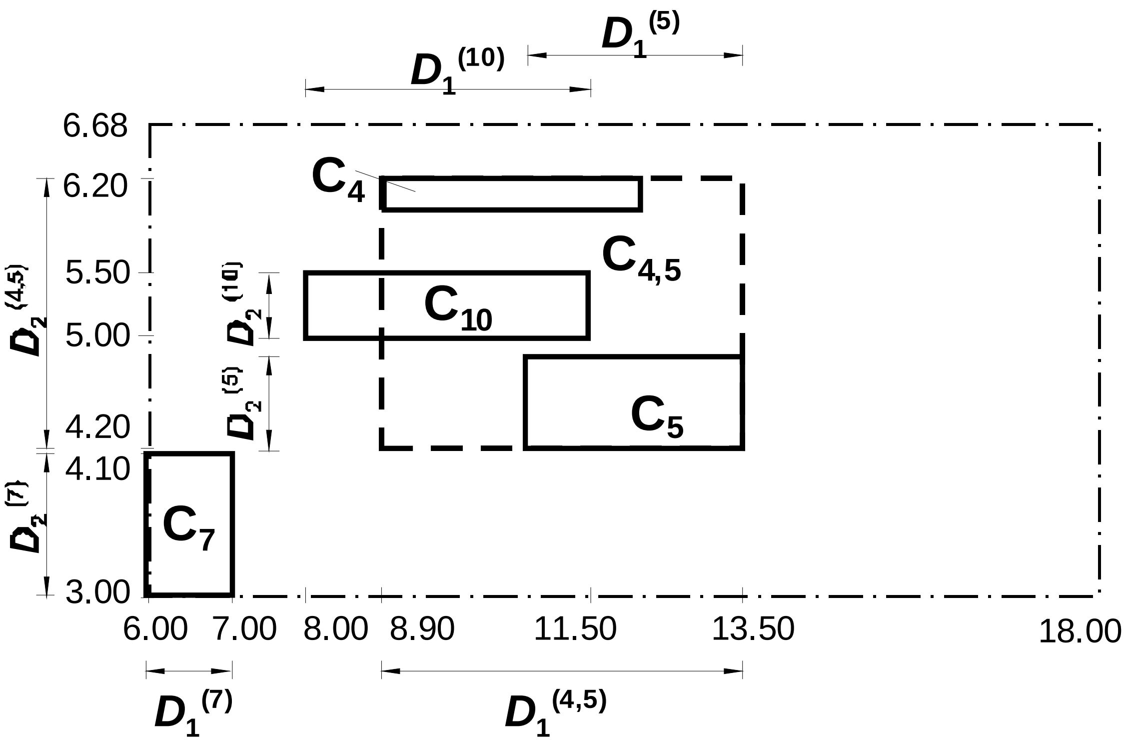


Example 1: Steel girder (3)

Cluster configuration in the space $\underline{D}^2$ based on k-medoid method



Clusters selected for alternative design variants



modified sets = alternative design variants

cluster C_5 : $D_1^{[5]}$, $D_2^{[5]}$

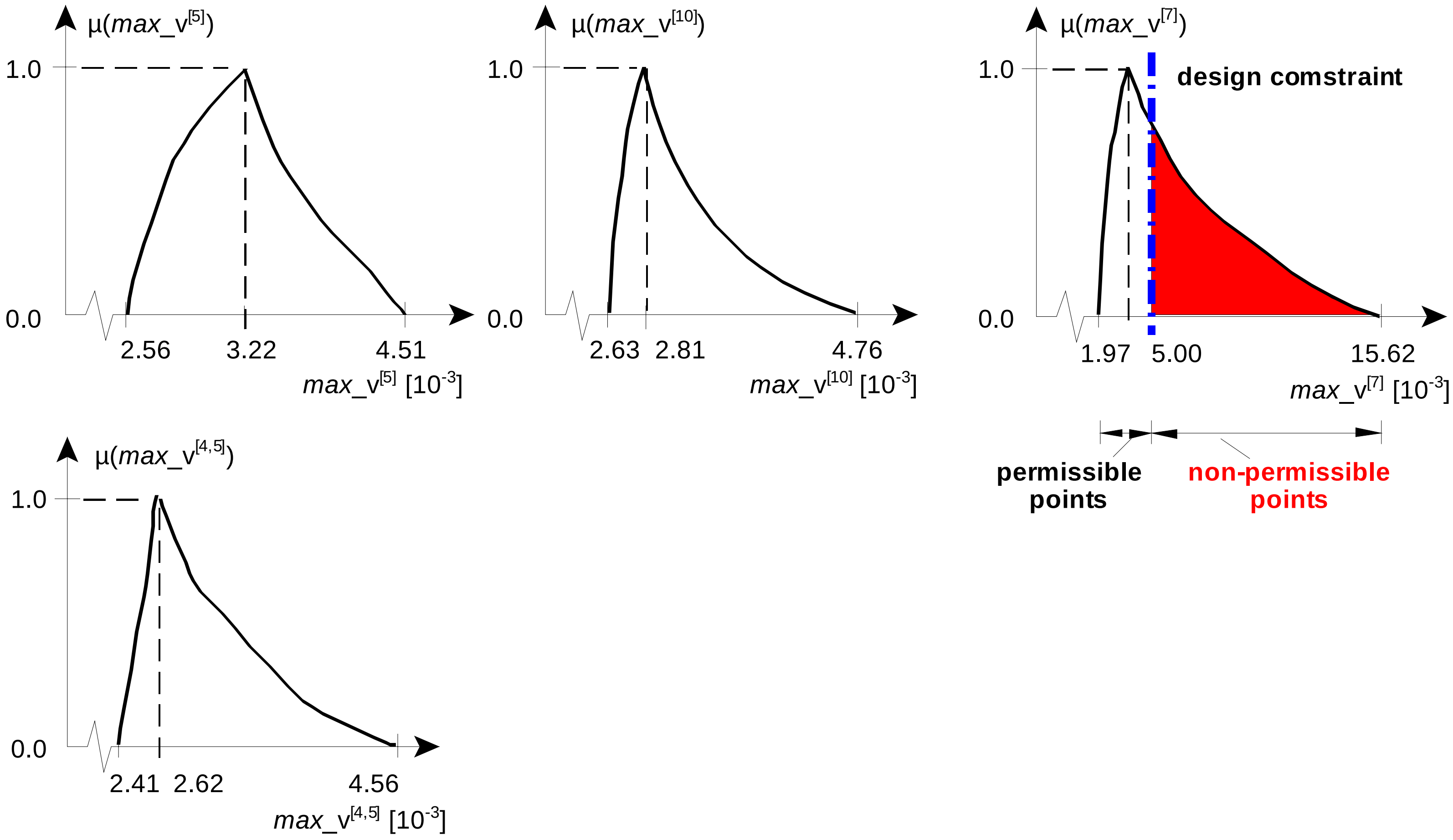
cluster C_7 : $D_1^{[7]}$, $D_2^{[7]}$

cluster C_{10} : $D_1^{[10]}$, $D_2^{[10]}$

cluster $C_{4,5}$: $D_1^{[4,5]}$, $D_2^{[4,5]}$

Example 1: Steel girder (5)

Verification of structural design alternatives yields modified fuzzy results



Example 1: Steel girder (6)

Computed design variants

Design variant I
(obtained from cluster 5)

$M^{[5]} = [10.8, 13.5]$ t

$l_1^{[5]} = [4.20, 4.80]$ m

Design variant II
(obtained from cluster 10)

$M^{[10]} = [8.00, 11.5]$ t

$l_1^{[10]} = [5.00, 5.50]$ m

Design variant III
(obtained from cluster 4,5)

$M^{[4,5]} = [8.90, 13.5]$ t

$l_1^{[4,5]} = [4.20, 6.20]$ m

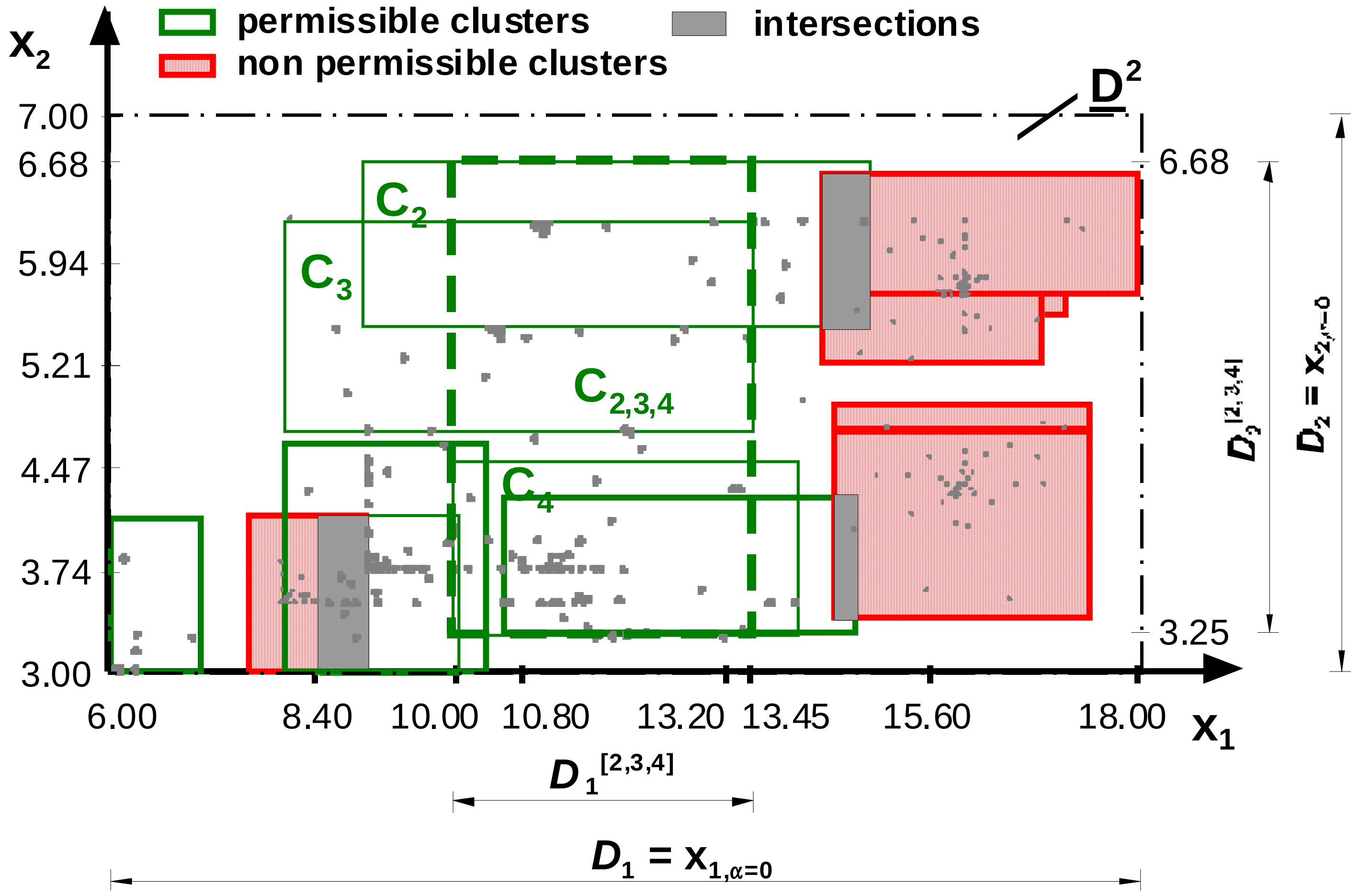
Example 1: Steel girder (7)

Assessment of the design variants

	defuzzification results		
	$max\_v_0^{[5]}$	$max\_v_o^{[10]}$	$max\_v_o^{[4,5]}$
centroid method	3.37	3.39 (max)	3.11 (min)
Chen method	4.00 (min)	4.60 (max)	4.30
level rank method	3.30 (max)	3.10	2.97 (min)
robustness measure B	0.03933 (max)	0.00458	0.00137 (min)

Example 1: Steel girder (8)

Cluster configuration in the space D^2 based on fuzzy cluster method



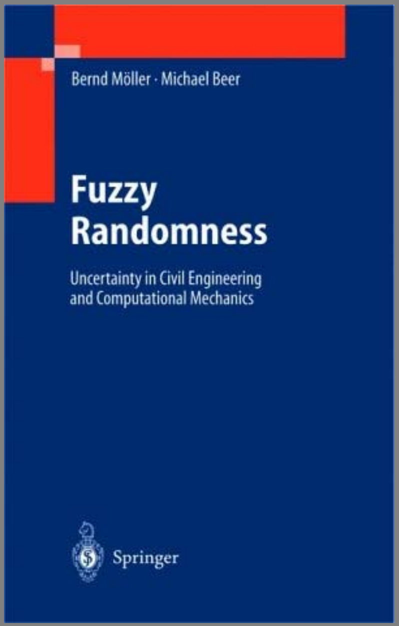
Recent research results about non-classical methods in uncertainty modeling

http://www.uncertainty-in-engineering.net

and

Fuzzy Randomness

B. Möller, M. Beer, Springer 2004



Thank you !