TECHNISCHE UNIVERSITÄT DRESDEN

Institute for Statics and Dynamics of Structures

Demolition of structures considering uncertainty

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Program

KW Thierbach,
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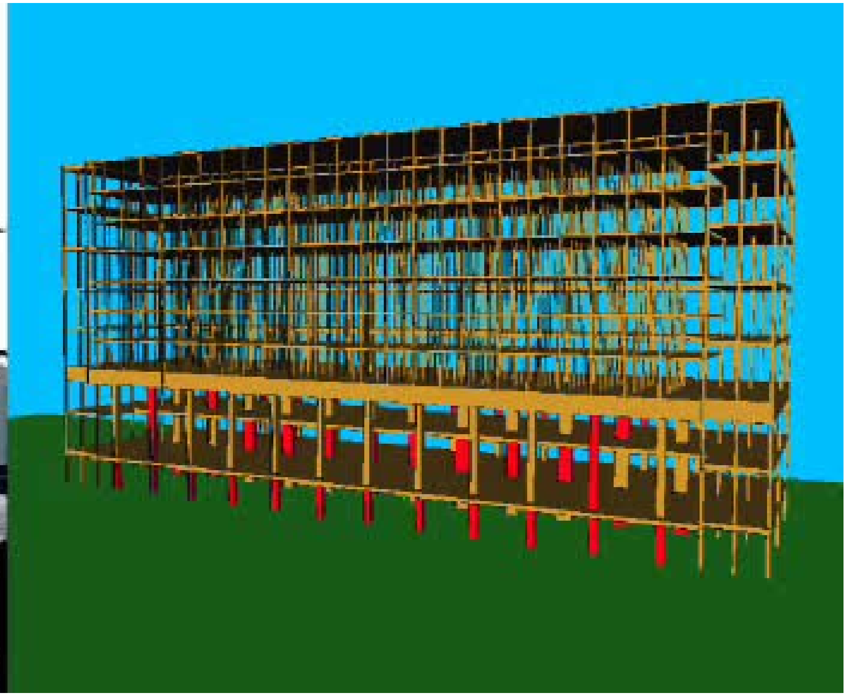
Introduction (1)



Introduction (2)

structural parameters

- geometry
- material
- failure zones

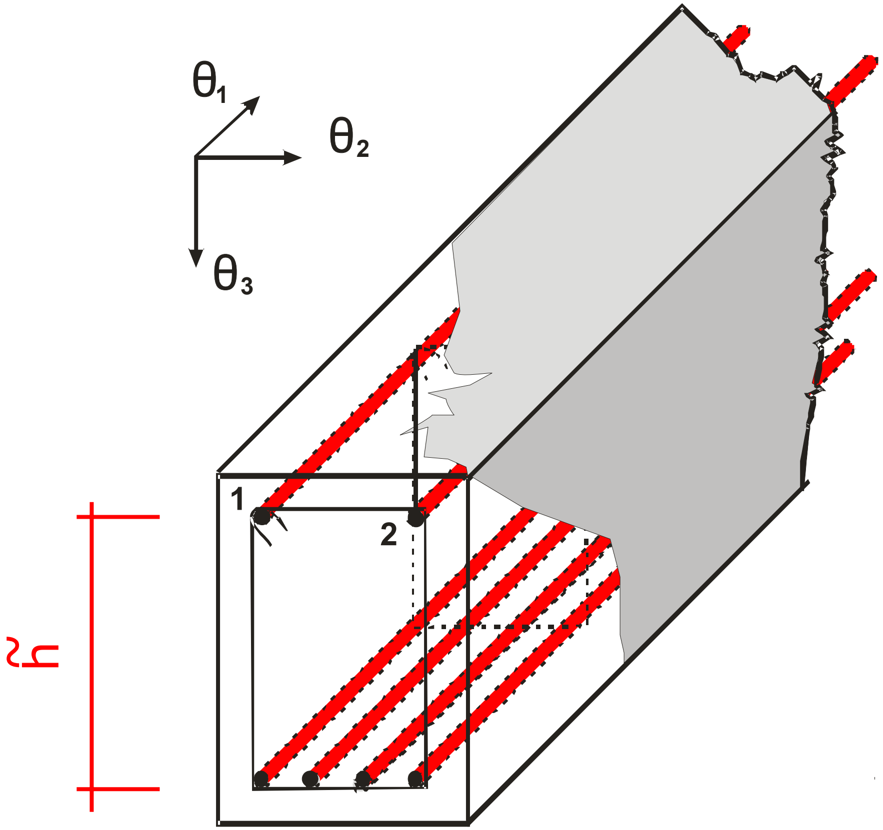
blasting parameters

- time of detonation
- detonation impacts
- grade of local structural damage

contact parameters

- friction
- damping

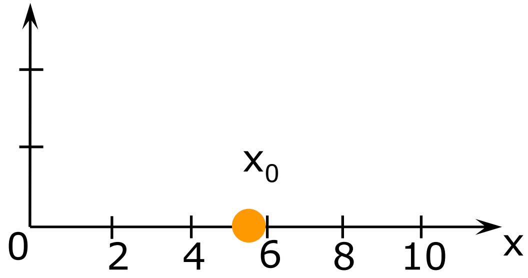
uncertain geometrical parameters



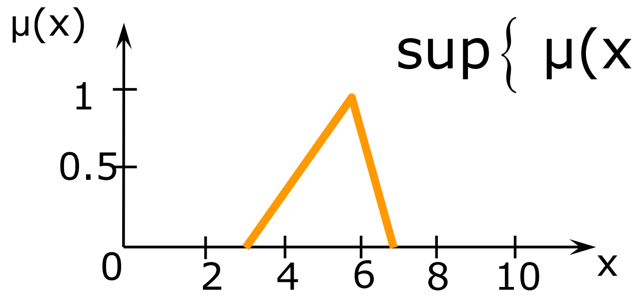
uncertain position of reinforcement

Data Models (1)

deterministic variable

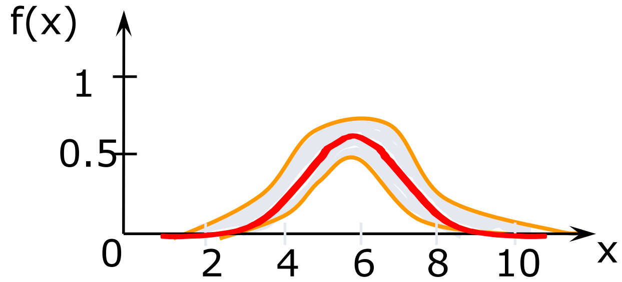


fuzzy variable



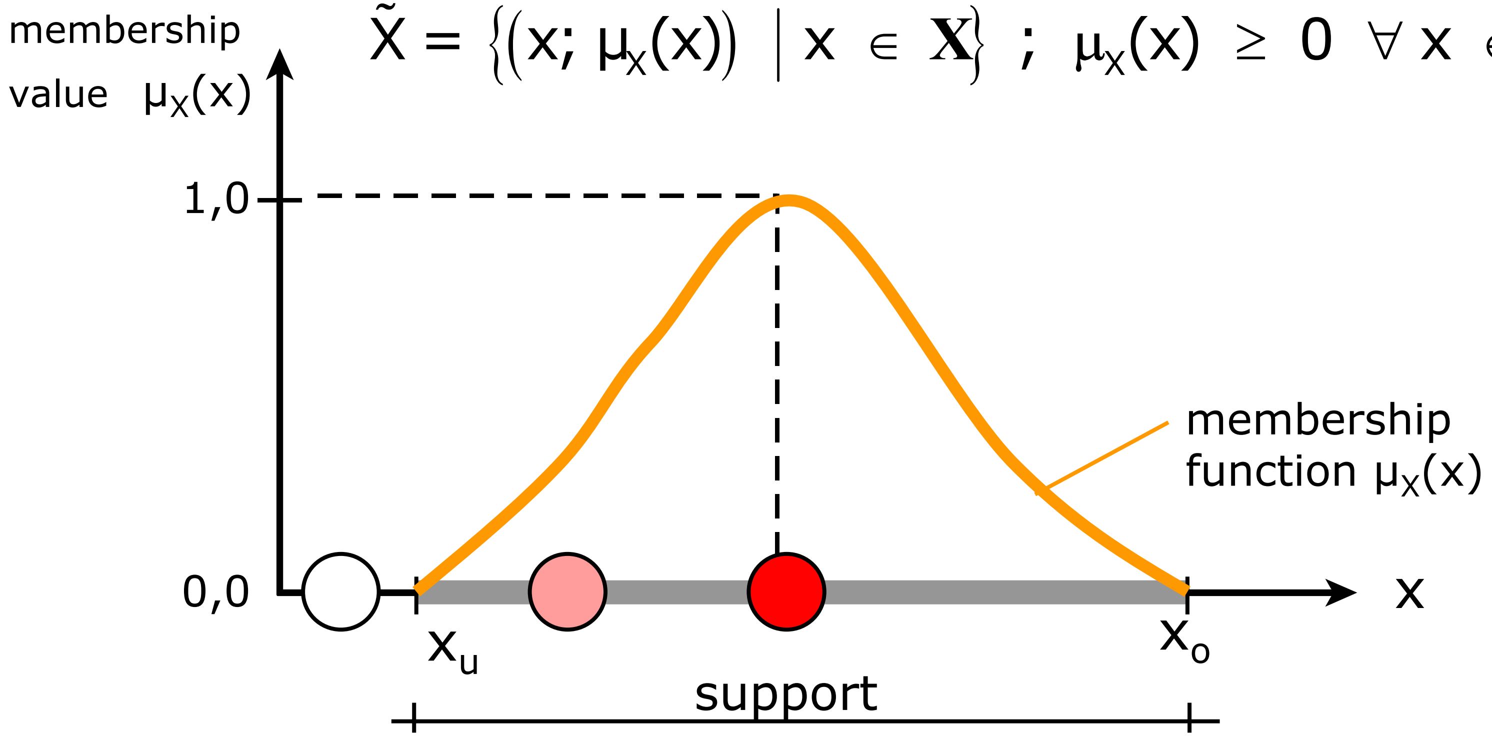
$$\sup\left\{ \mu(x) \mid x \in R \right\} = 1$$

fuzzy random variable



fuzzy variable $\tilde{X}$

$$\tilde{X} = \{(x; \mu_X(x)) \mid x \in \mathbf{X}\} \;;\; \mu_X(x) \geq 0 \;\; \forall x \in \mathbf{X}$$



a fuzzy number possesses exactly one x with the membership value $\mu_X(x) = 1$

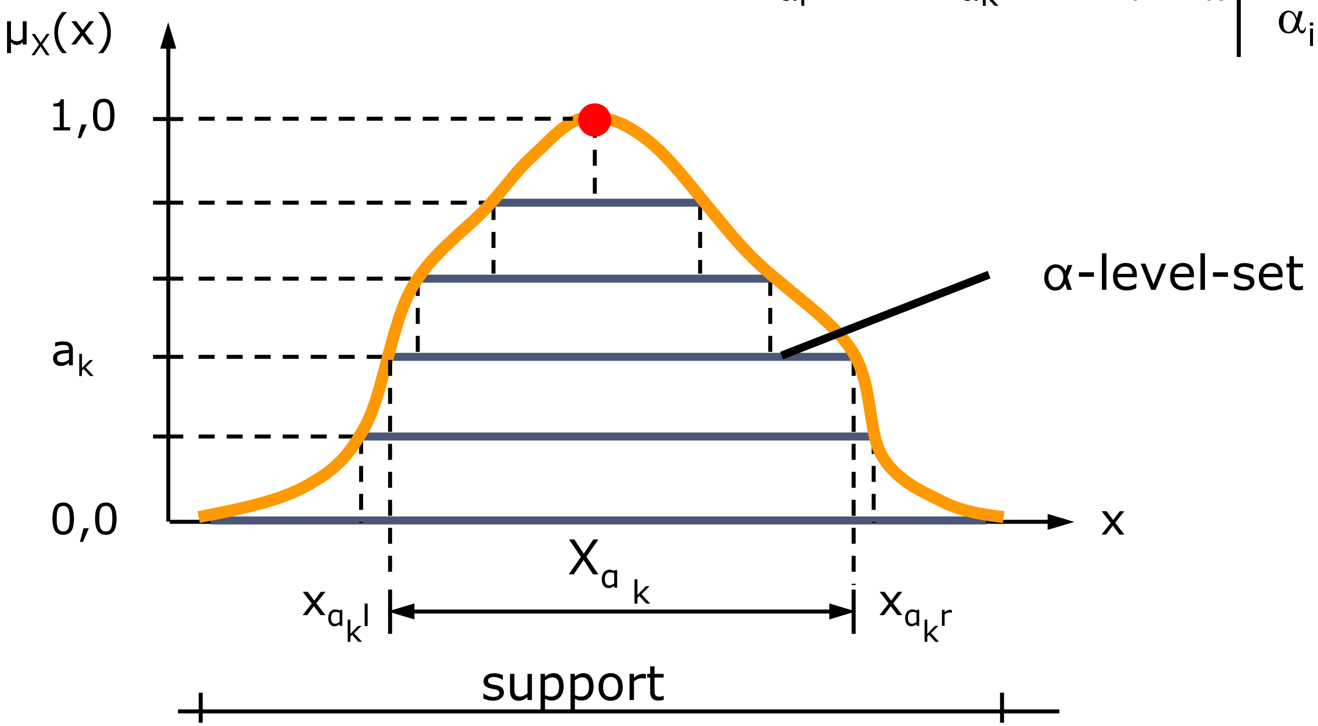
α-level-set

$$X_{\alpha_k} = \{x \in X \mid \mu_x(x) \geq \alpha_k\}$$

α-discretization

$$\tilde{X} = \{(X_\alpha; \mu(X_\alpha))\}$$

$$X_{\alpha_i} \subseteq X_{\alpha_k} \quad \forall\, \alpha_i, \alpha_k \;\middle|\; \begin{array}{l} \alpha_i \geq \alpha_k \\ \alpha_i; \alpha_k \in (0,1] \end{array}$$



Data Models (4) – Fuzzy functions

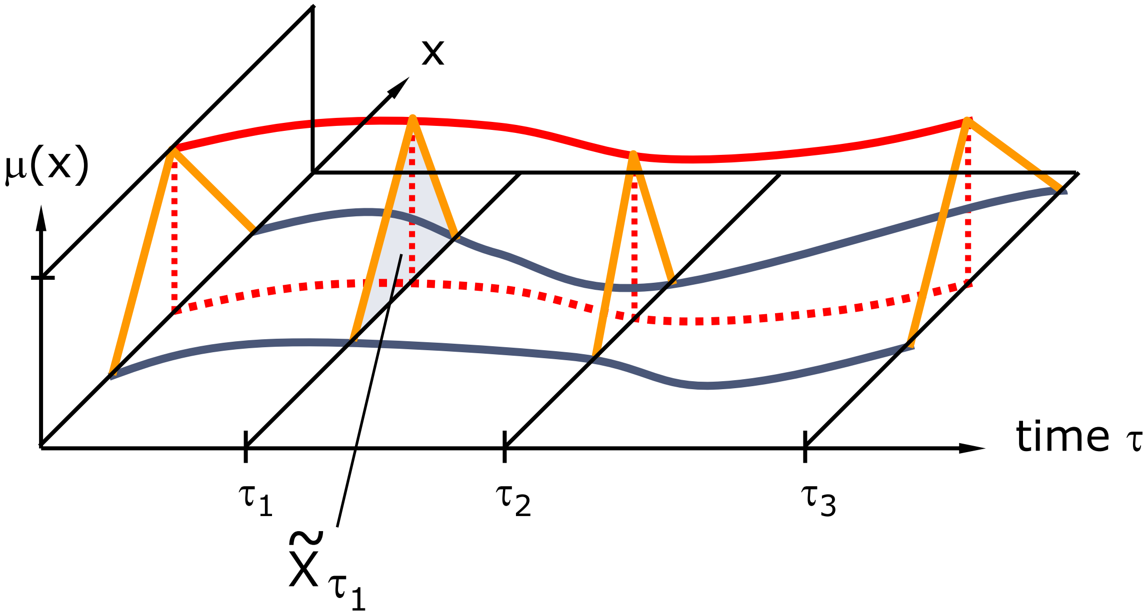
fuzzy function: $\tilde{\underline{x}}(\underline{t})$: T $\rightsquigarrow$ F(X)

parameter space — set of fuzzy variables

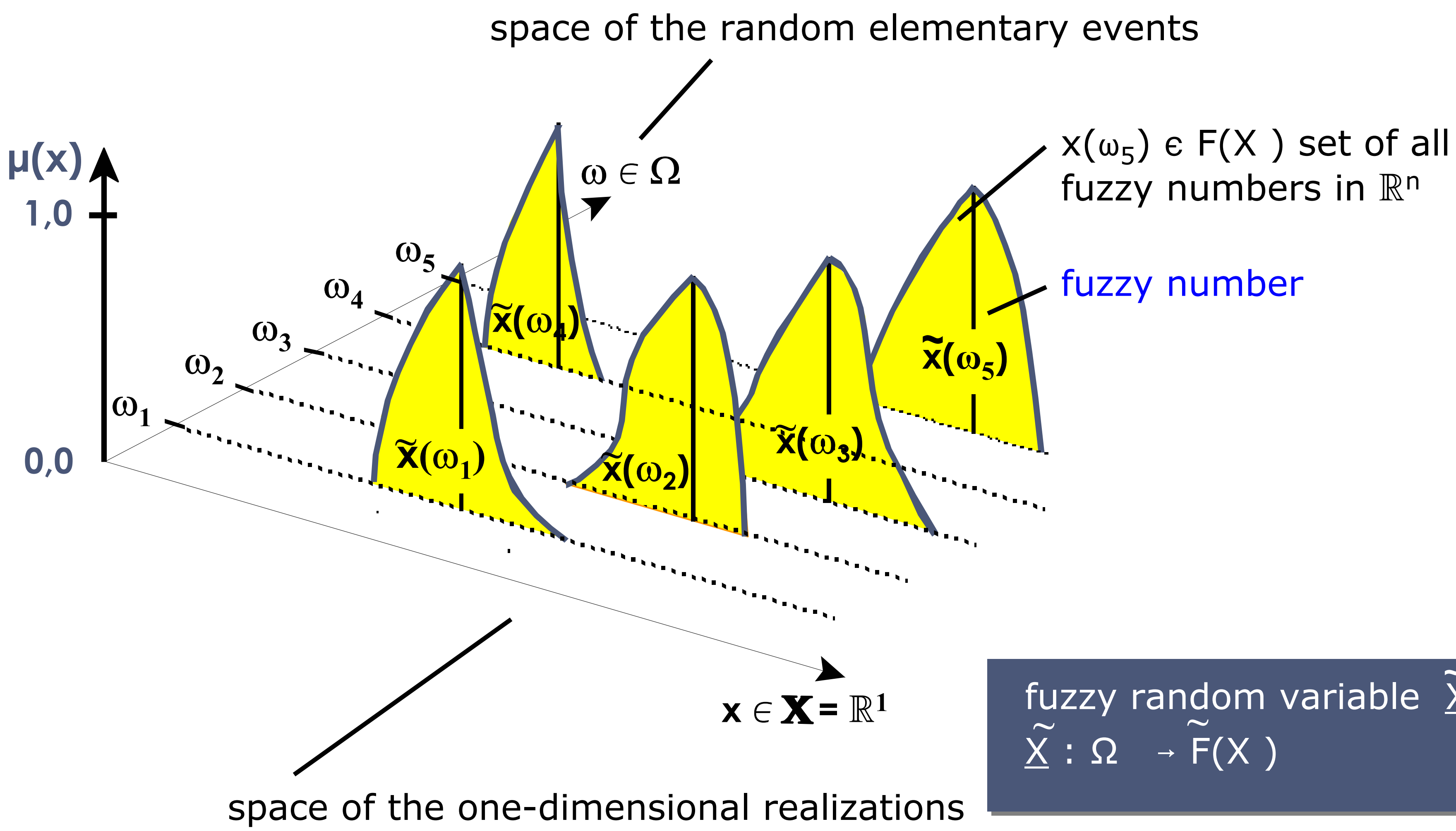
$\underline{t} = (\underline{\theta}, \tau, \underline{\varphi})$

$$\tilde{\underline{x}}(\underline{t}) = \left\{ \tilde{x}_t = \tilde{x}(\underline{t}) \ \forall \ \underline{t} \,\middle|\, \underline{t} \in T \right\}$$

fuzzy process: $\tilde{\underline{x}}(\tau) = \left\{ \tilde{x}_\tau = \tilde{x}(\tau) \ \forall \ \tau \,\middle|\, \tau \in T \right\}$

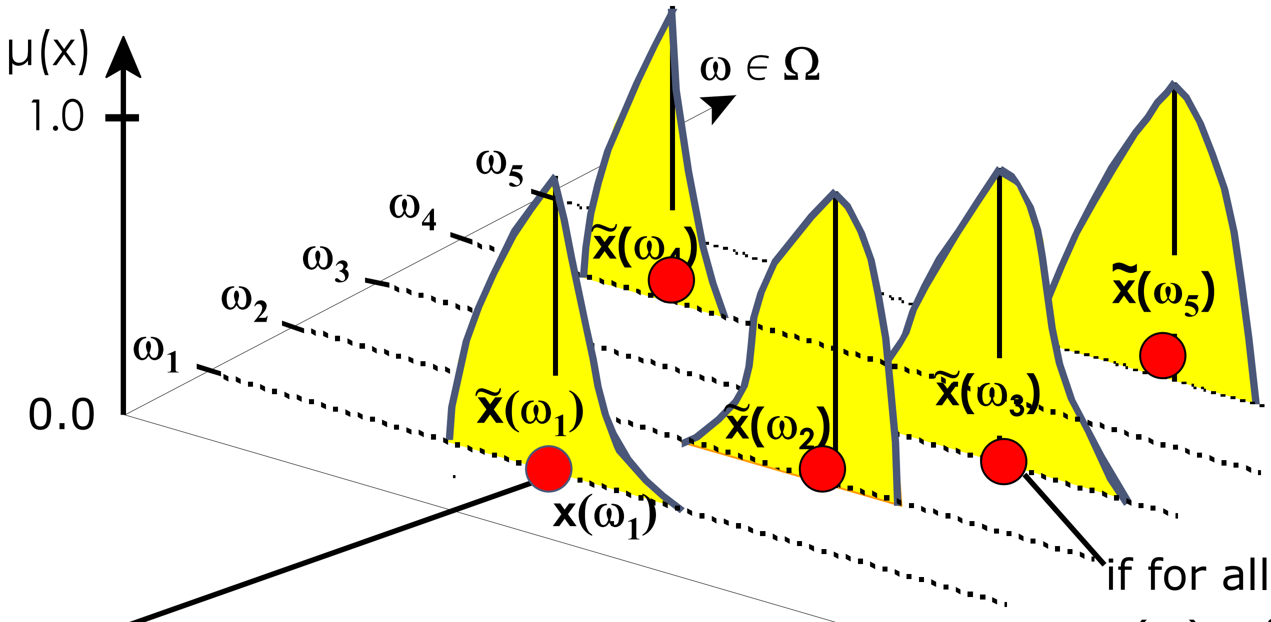


Data Models (6) – Fuzzy Randomness



fuzzy random variable $\tilde{X}$

$\tilde{X} : \Omega \rightarrow \tilde{F}(X)$



$x \in \mathbf{X} = \mathbb{R}^1$

$x(\omega_1)$: realization of the real random variable X

$x(\omega_1) \in \tilde{x}(\omega_1)$

if for all $x(\omega_i)$ holds:
$x(\omega_i) \in \tilde{x}(\omega_i)$,
then: $\underline{x}$ constitute one **original X_j** of $\tilde{\mathbf{X}}$

an original $\mathbf{X_j}$ has the porperty of a real random variable **X**

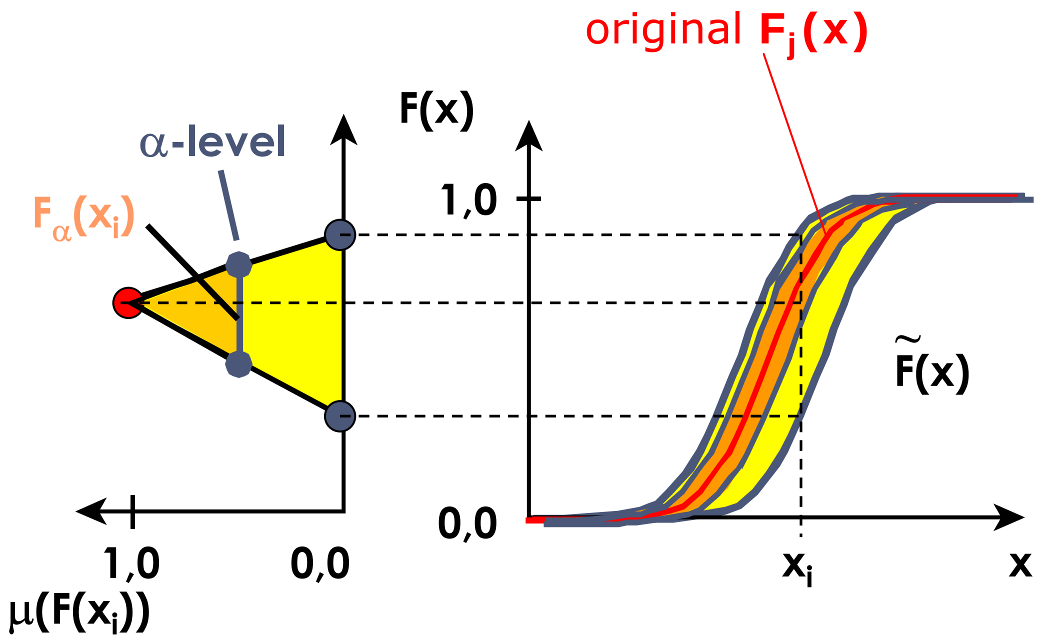
$\underline{\tilde{X}}$:= fuzzy set of all originals $\underline{X}_j$

e.g. Gumbel

$$\tilde{F}(x) = \exp\left(-\exp\left(-\tilde{s}_1 \cdot (x-\tilde{s}_2)\right)\right)$$

$$\tilde{s}_1 = \frac{\pi}{\tilde{\sigma}_X \cdot \sqrt{6}}; \quad \tilde{s}_2 = \tilde{m}_X - 0{,}45 \cdot \tilde{\sigma}_X$$

$$\tilde{F}(x) = F(\underline{\tilde{s}},\ x)$$



fuzzy probability distribution function

Data Models – Fuzzy Random Functions

given: set of fuzzy random variables $\tilde{X}$ at points $\underline{t} \in \underline{T}$

with $\underline{t} = \{\tau, \underline{\theta}\}$, τ time

$\underline{\theta} = \{\theta_1, \theta_2, \theta_3\}$ spatial coordinates

definition: A fuzzy random function $\tilde{X}(\underline{t})$ is the set of fuzzy random variables $\tilde{X}_t$ in $\underline{T}$

$$\tilde{X}(\underline{t}) = \left\{ \tilde{X}_t = \tilde{X}(\underline{t}) \; \forall \underline{t} \; \middle| \; \underline{t} \in \underline{T} \right\}$$

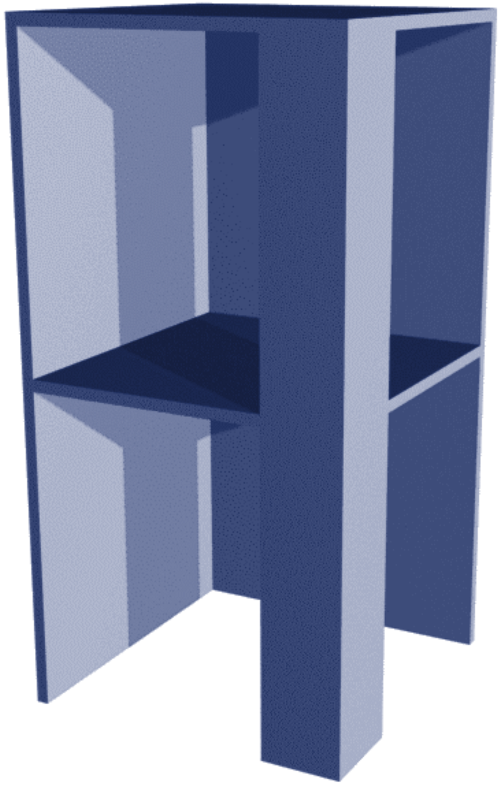
$$X(\underline{t}) : \; T \times \Omega \rightarrow F(X)$$

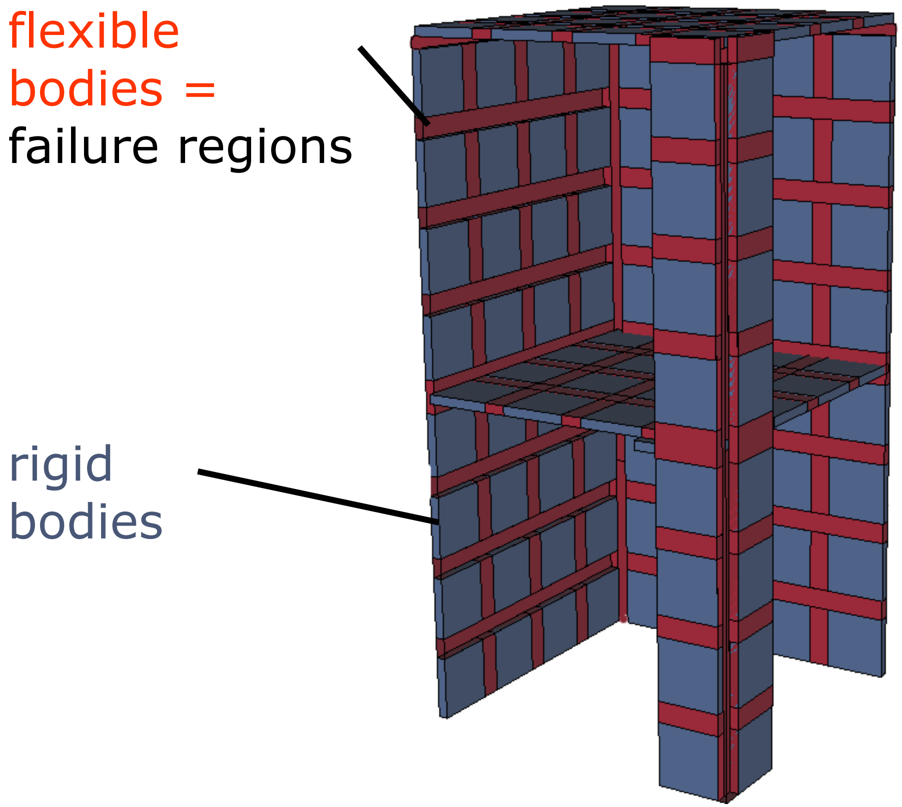
special cases:

1. no randomness: $\tilde{X}(\underline{t}) \rightarrow \tilde{x}(\underline{t})$ fuzzy function
2. no fuzziness: $\tilde{X}(\underline{t}) \rightarrow X(w)$ random function
3. for fixed t : $\tilde{X}(\underline{t}) \rightarrow \tilde{X}(\underline{q})$ fuzzy random field

Numerical Model (1)

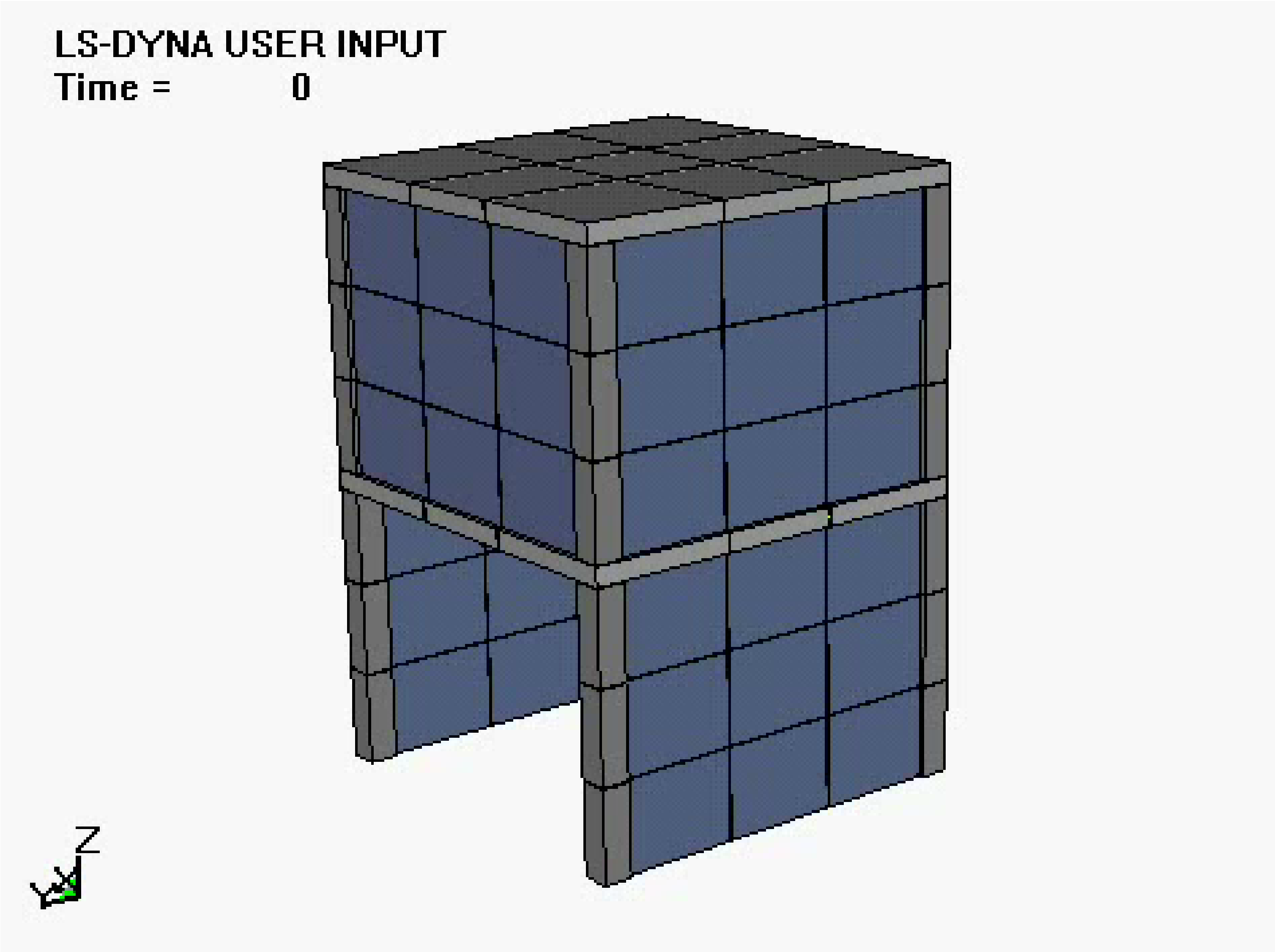
Numerical Model (2)



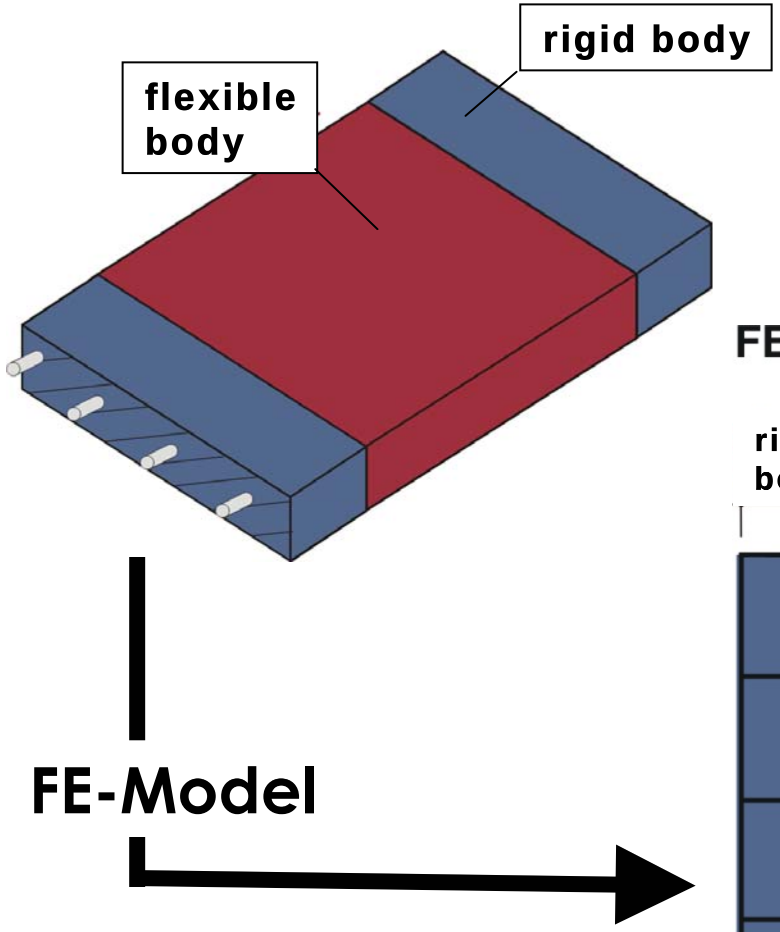


problem of multibody dynamics
with data uncertainties

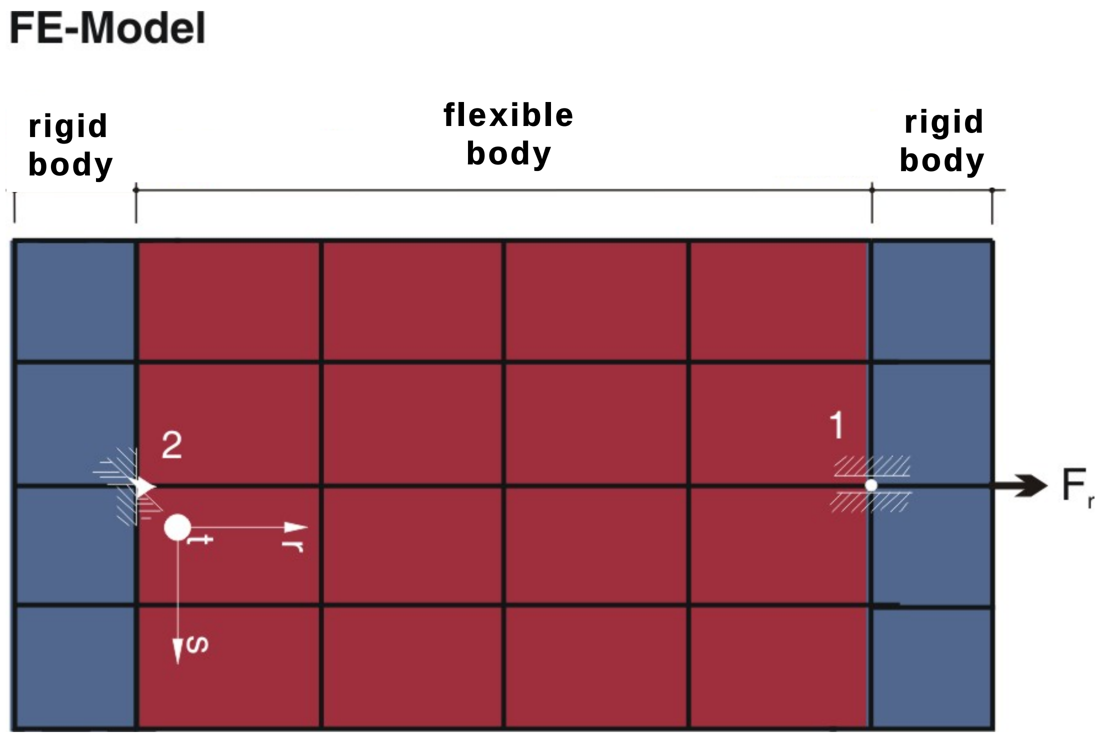
Numerical Model (3)



Numerical Model (4)



considering the **nonlinear behavior** of **reinforcement concrete**



Numerical Model (5)

$$
\begin{bmatrix} F_1(1) \\ F_2(1) \\ F_3(1) \\ M_1(1) \\ M_2(1) \\ M_3(1) \\ F_1(2) \\ F_2(2) \\ F_3(2) \\ M_1(2) \\ M_2(2) \\ M_3(2) \end{bmatrix} = \begin{bmatrix} k_{11}(1,1) & \cdots & k_{11}(1,6) & k_{12}(1,1) & \cdots & k_{12}(1,6) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{11}(6,1) & \cdots & k_{11}(6,6) & k_{12}(6,1) & \cdots & k_{12}(6,6) \\ k_{21}(1,1) & \cdots & k_{21}(1,6) & k_{22}(1,1) & \cdots & k_{22}(1,6) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ k_{21}(6,1) & \cdots & k_{21}(6,6) & k_{22}(6,1) & \cdots & k_{22}(6,6) \end{bmatrix} \bullet \begin{bmatrix} v_1(1) \\ v_2(1) \\ v_3(1) \\ \varphi_1(1) \\ \varphi_2(1) \\ \varphi_3(1) \\ v_1(2) \\ v_2(2) \\ v_3(2) \\ \varphi_1(2) \\ \varphi_2(2) \\ \varphi_3(2) \end{bmatrix}
$$

- nonlinear relations between forces and displacements

Objective: A-priori-computation of force-deformation relationship

Hypothesis: diagonal element >> off diagonal element

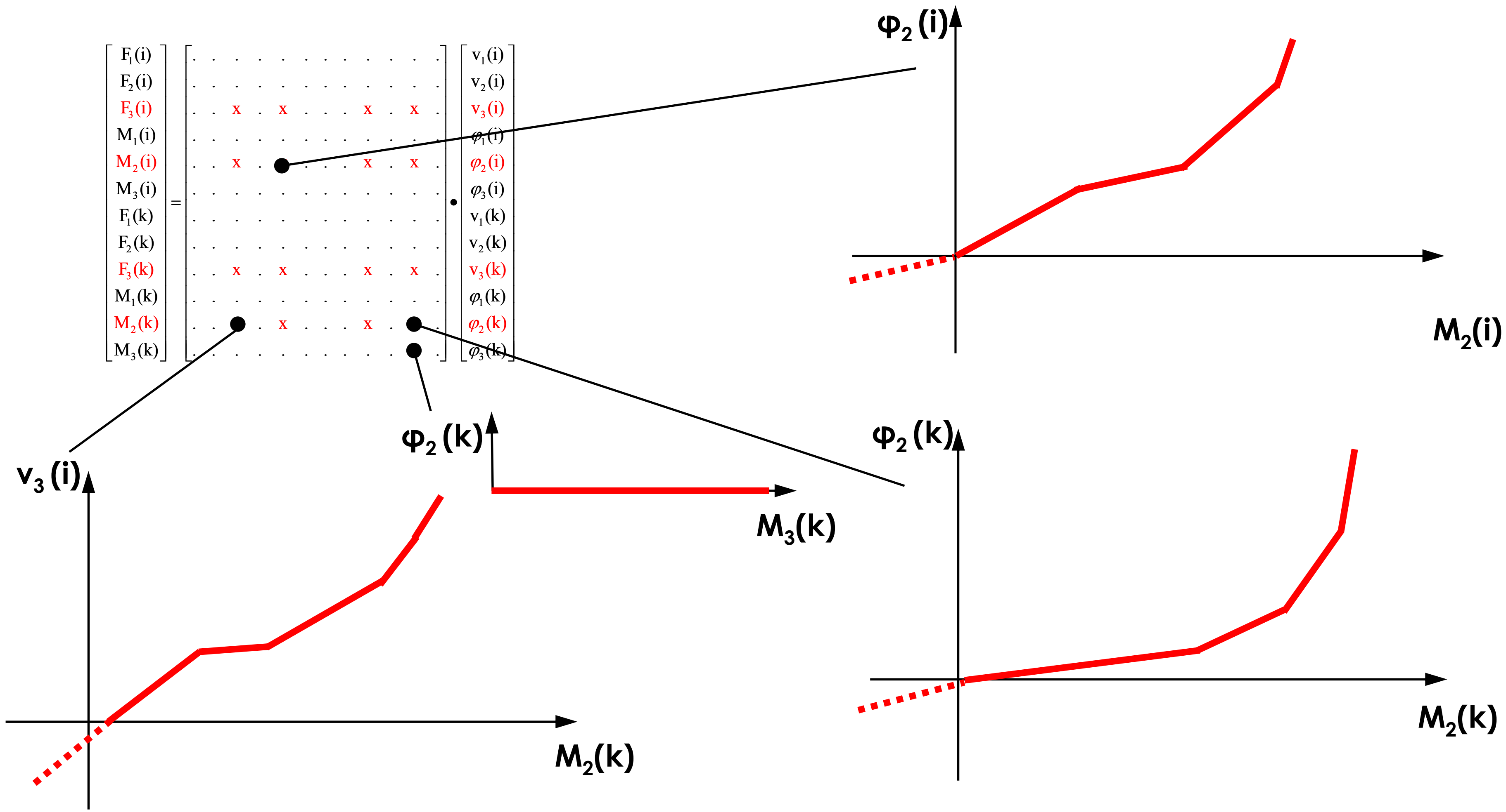
Decoupled system of equations:

$$F_i = k_i(v_i) \cdot v_i \qquad \text{bzw.} \qquad v_i = \frac{1}{k_i(v_i)} \cdot F_i$$

Spring element:

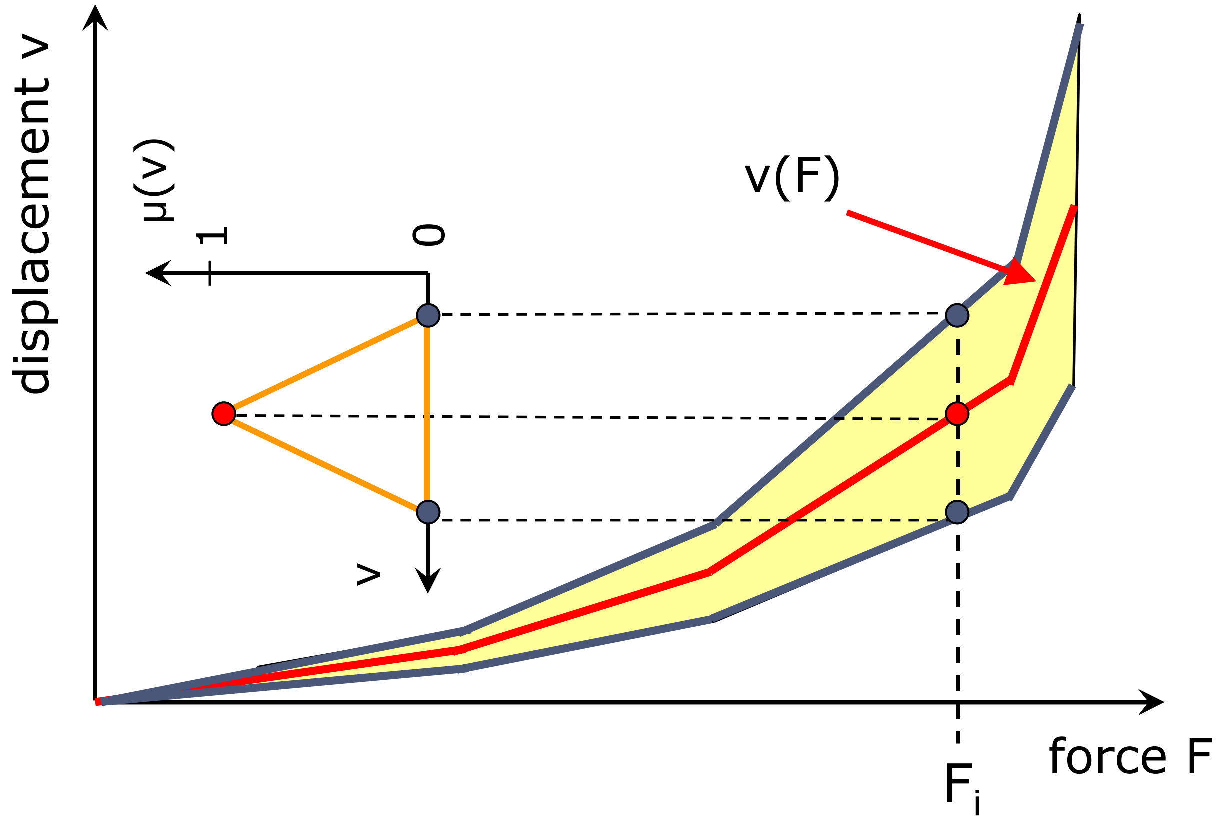
Numerical Model (6)

force deformation realation v(F)

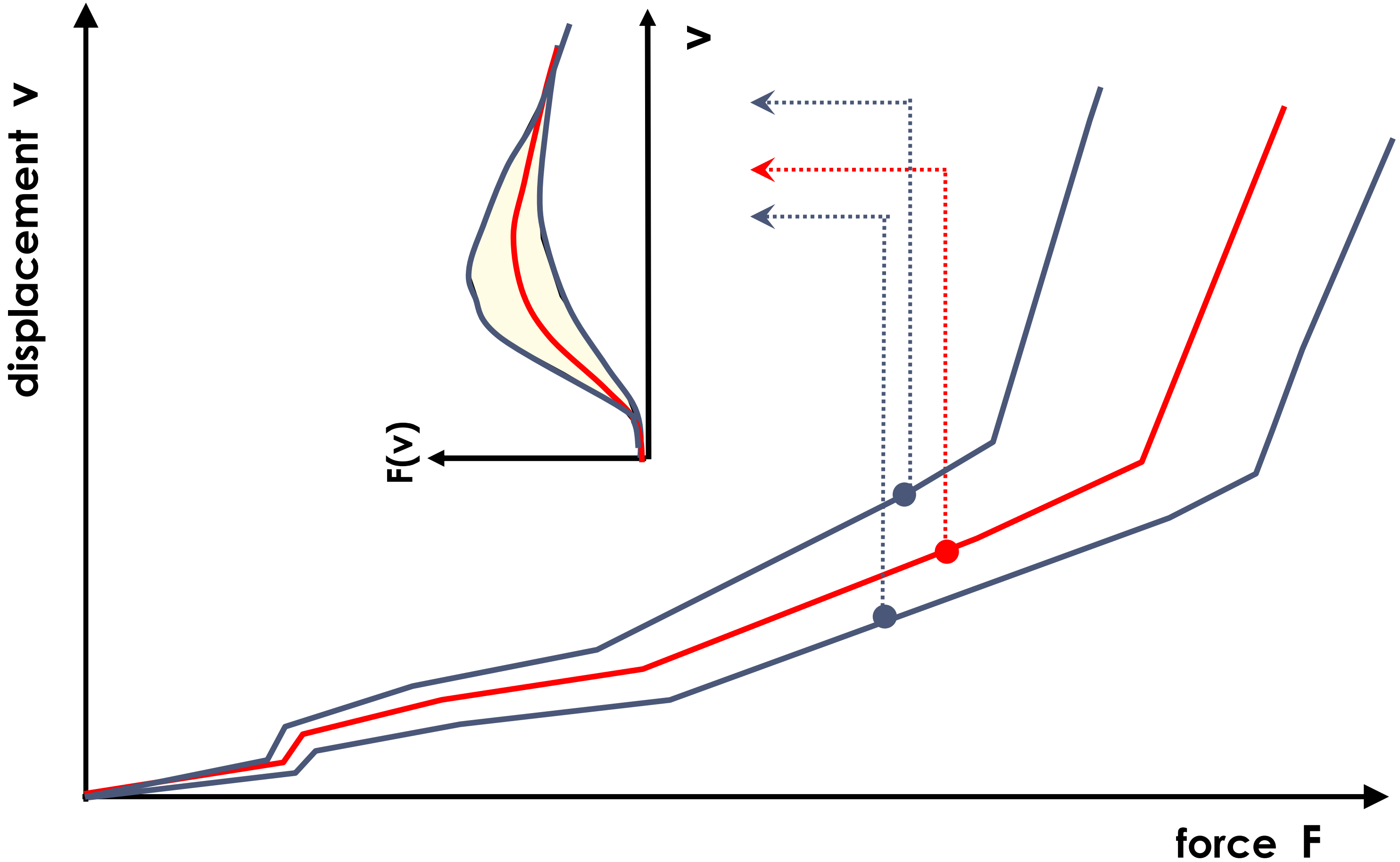


Numerical Model (6) – Fuzzy Function

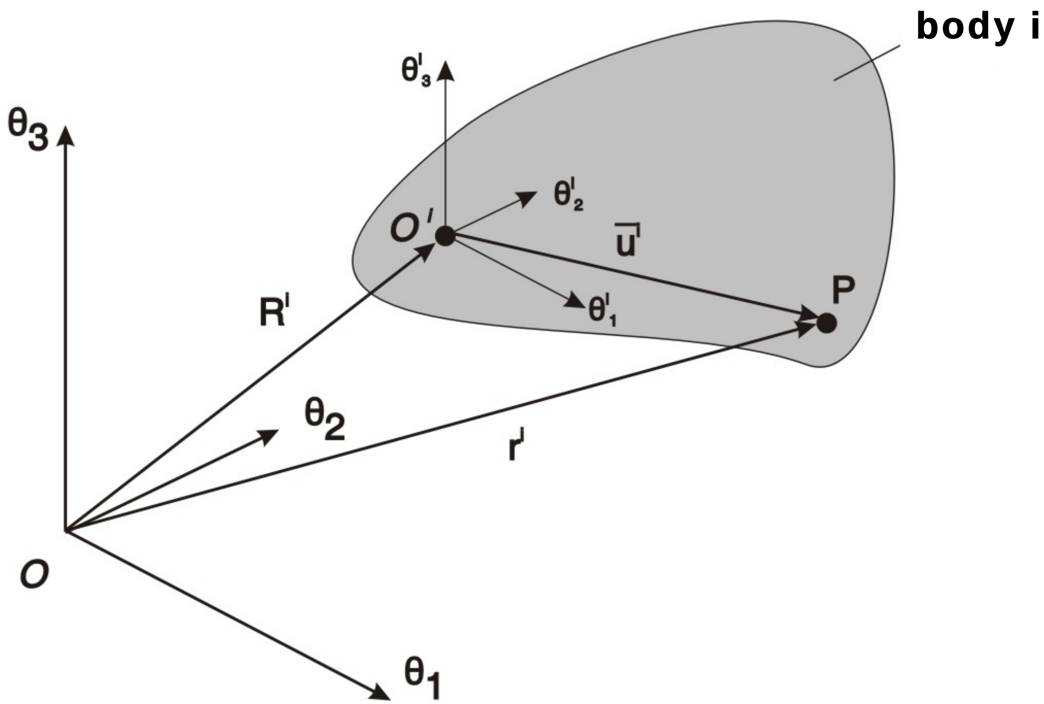
fuzzy function: $\tilde{v} = \tilde{v}(F)$



fuzzy random function $\tilde{v} = \tilde{v}(F)$



orientation of the rigid body i in R^3



$$\underline{q}_r^{\,i} = \left[\underline{R}^{i\,T} \;\; \underline{\theta}^{i\,T} \right]^T$$

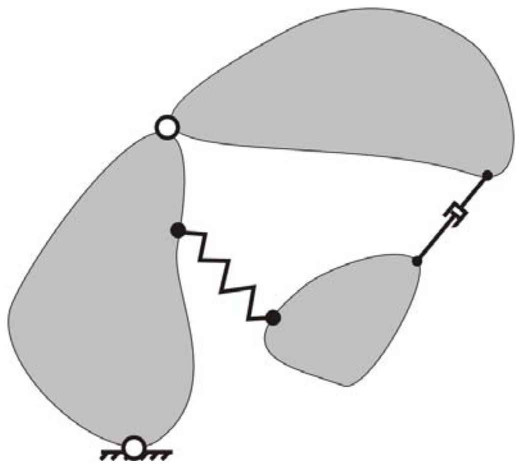
$$\underline{r}^i = \underline{R}^i + \underline{A}^i \underline{\bar{u}}^i$$

$\underline{A}$ transformation matrix

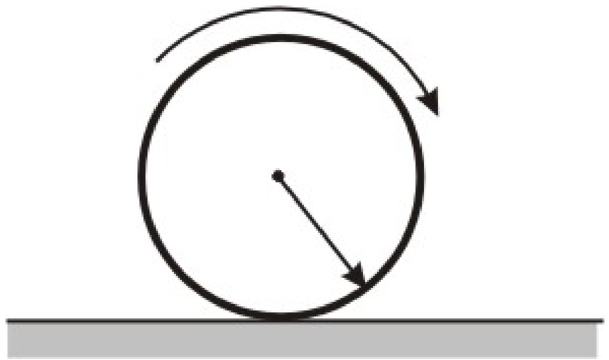
kinematical constraints

$$\underline{C}_{qr} = \underline{C}(\underline{q}_r, t) = \left[C_1(\underline{q}_r, t), C_2(\underline{q}_r, t), \dots, C_{nc}(\underline{q}_r, t) \right] = 0$$

n_b rigid bodies: $\underline{q}_r = \left[\underline{q}_r^{\,1}, \underline{q}_r^{\,2}, \dots, \underline{q}_r^{\,nb} \right]^T$



holonomic constraints



none holonomic constraints

Lagrangian equation of motion

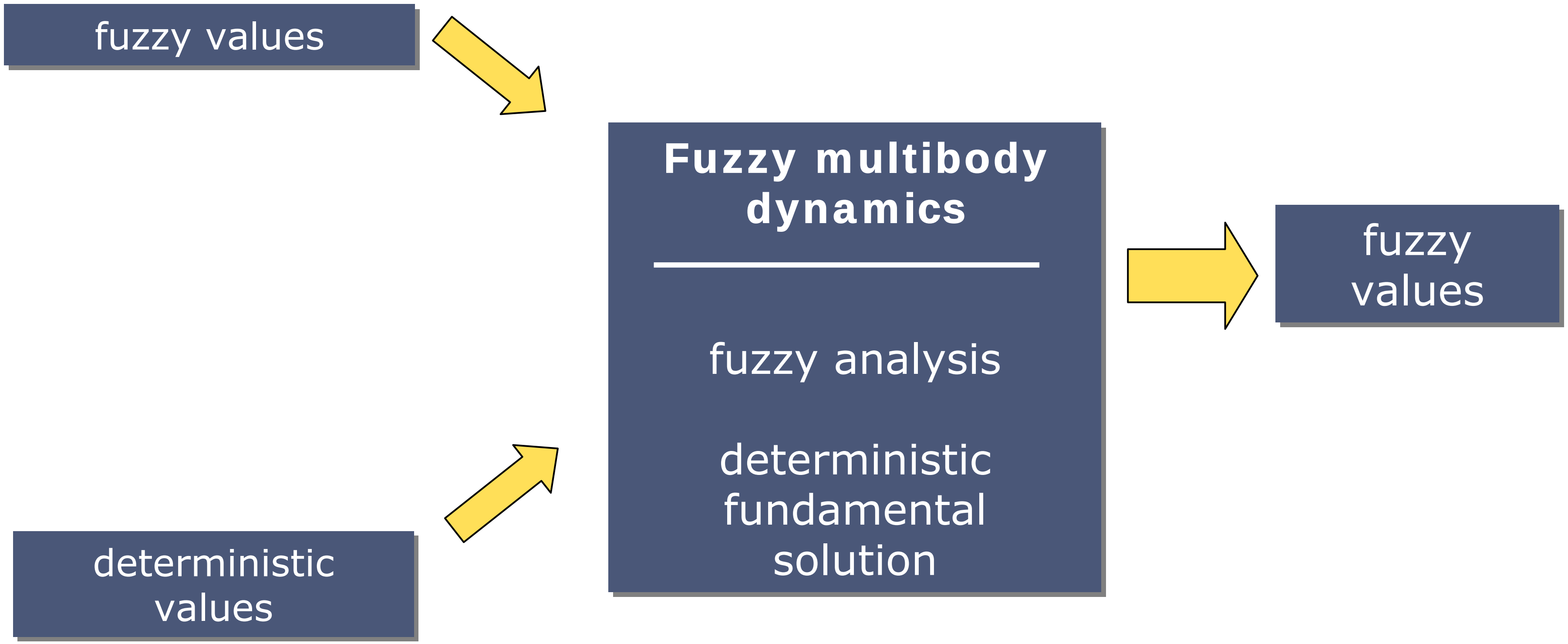
rigid bodies only:

$$\underline{M}\ddot{\underline{q}}_r + \underline{C}_{qr}^T \underline{\lambda} = \underline{Q}_e + \underline{Q}_v$$

rigid and flexible bodies:

$$\begin{bmatrix} \underline{m}_{rr} & \underline{m}_{rf} \\ \underline{m}_{fr} & \underline{m}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\underline{q}}_r \\ \ddot{\underline{q}}_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \underline{K}_{ff} \end{bmatrix} \begin{bmatrix} \underline{q}_r \\ \underline{q}_f \end{bmatrix} + \begin{bmatrix} \underline{C}_{qr}^T \\ \underline{C}_{qf}^T \end{bmatrix} \underline{\lambda} = \begin{bmatrix} (\underline{Q}_r)_e \\ (\underline{Q}_f)_e \end{bmatrix} + \begin{bmatrix} (\underline{Q}_r)_v \\ (\underline{Q}_f)_v \end{bmatrix}$$

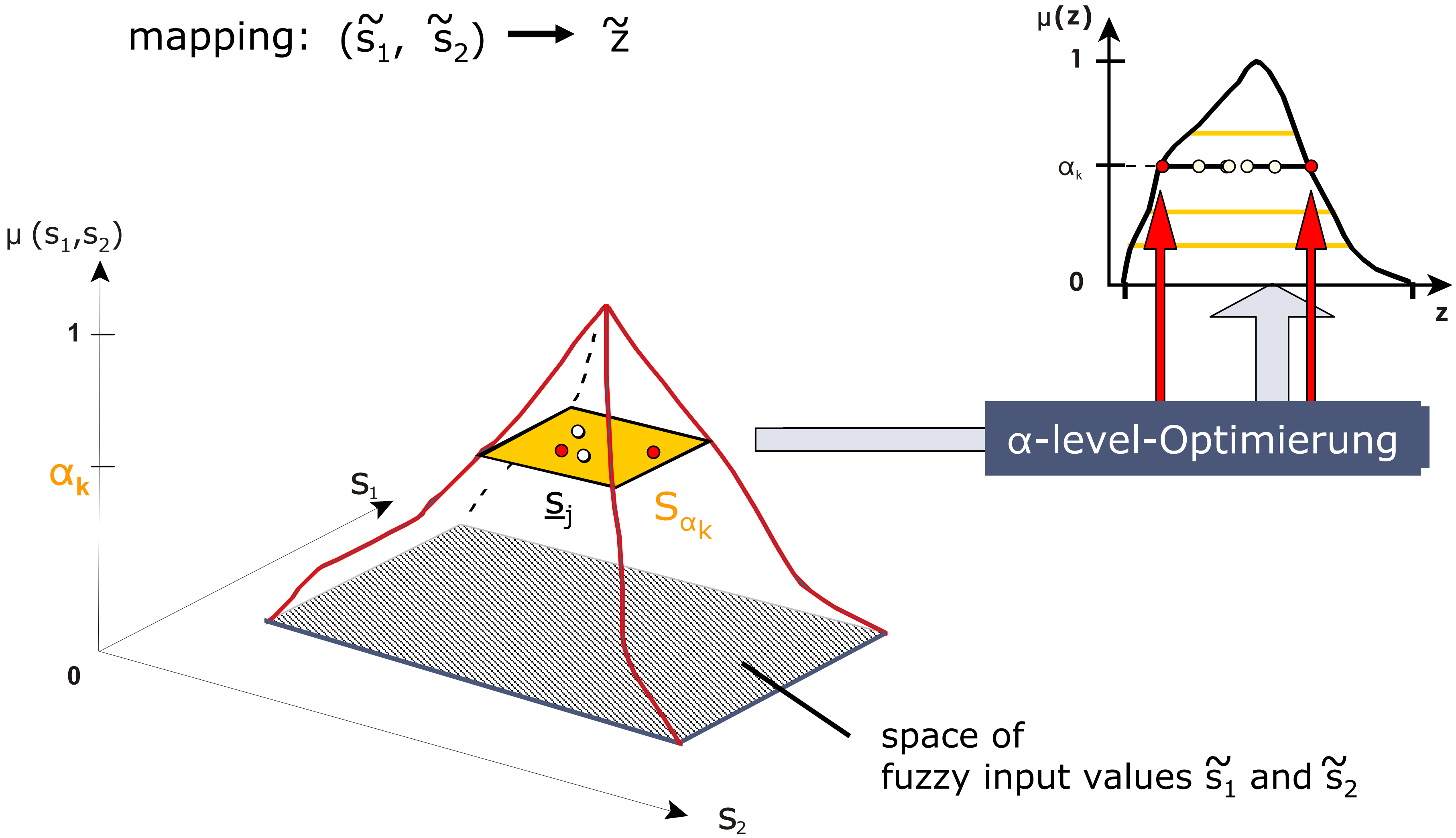
Fuzzy Multibody Dynamics (1)



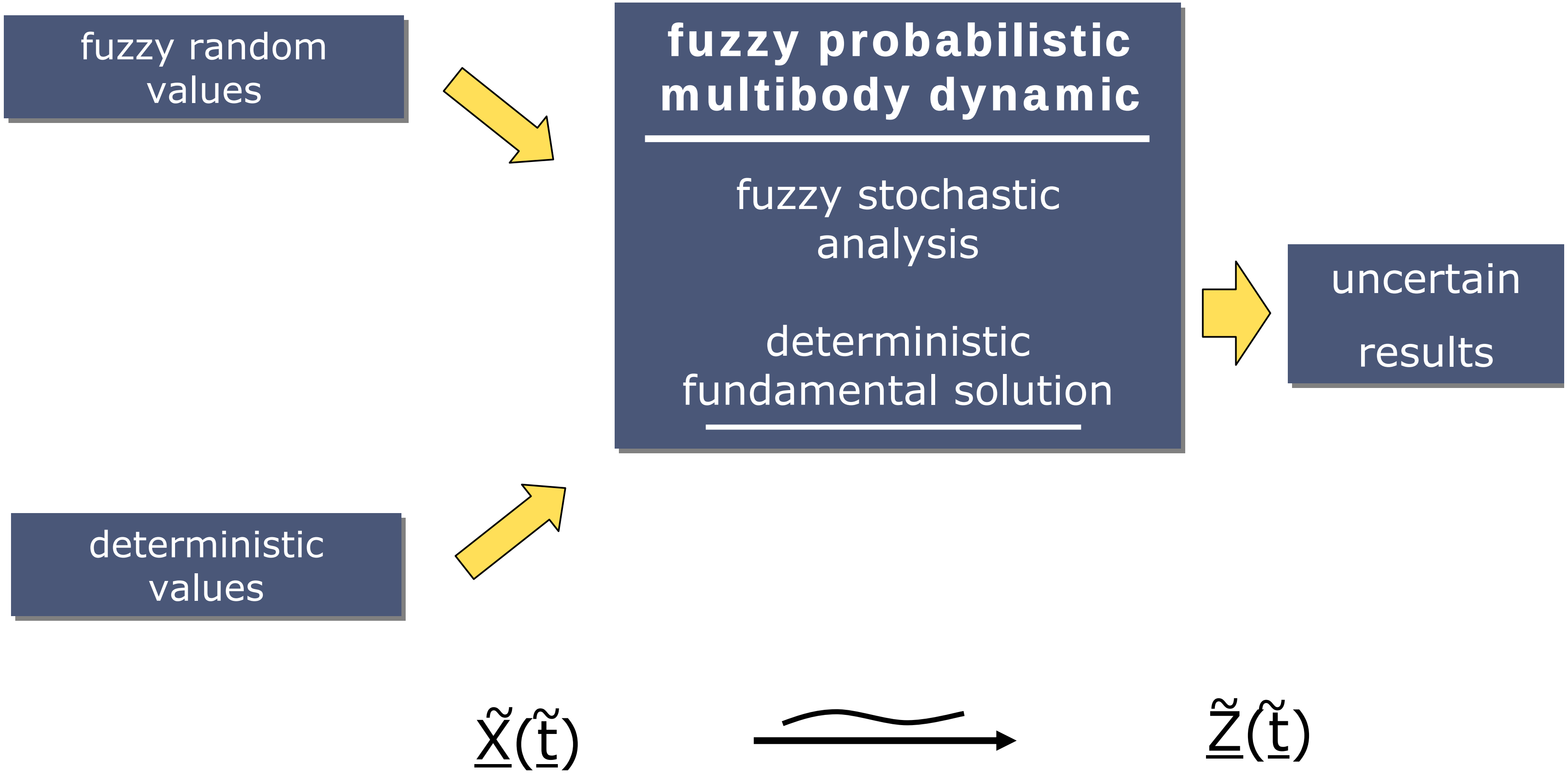
$$\underline{\tilde{X}}(\underline{\tilde{t}}) \xrightarrow{\;\sim\;} \underline{\tilde{Z}}(\underline{\tilde{t}})$$

Fuzzy Multibody Dynamics (2)

mapping: $(\tilde{s}_1, \tilde{s}_2) \longrightarrow \tilde{z}$



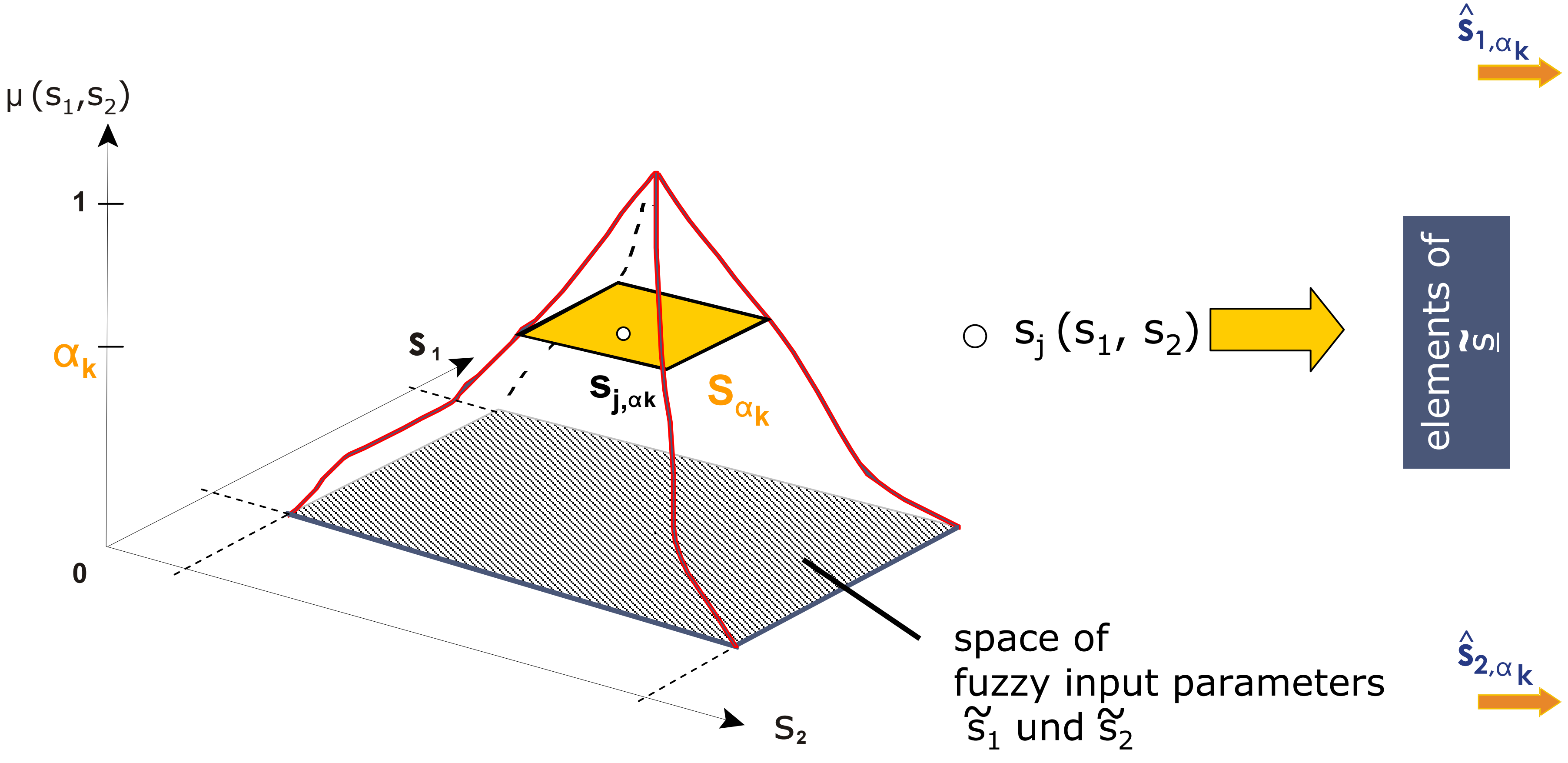
Fuzzy Probabilistic Multibody Dynamics (1)



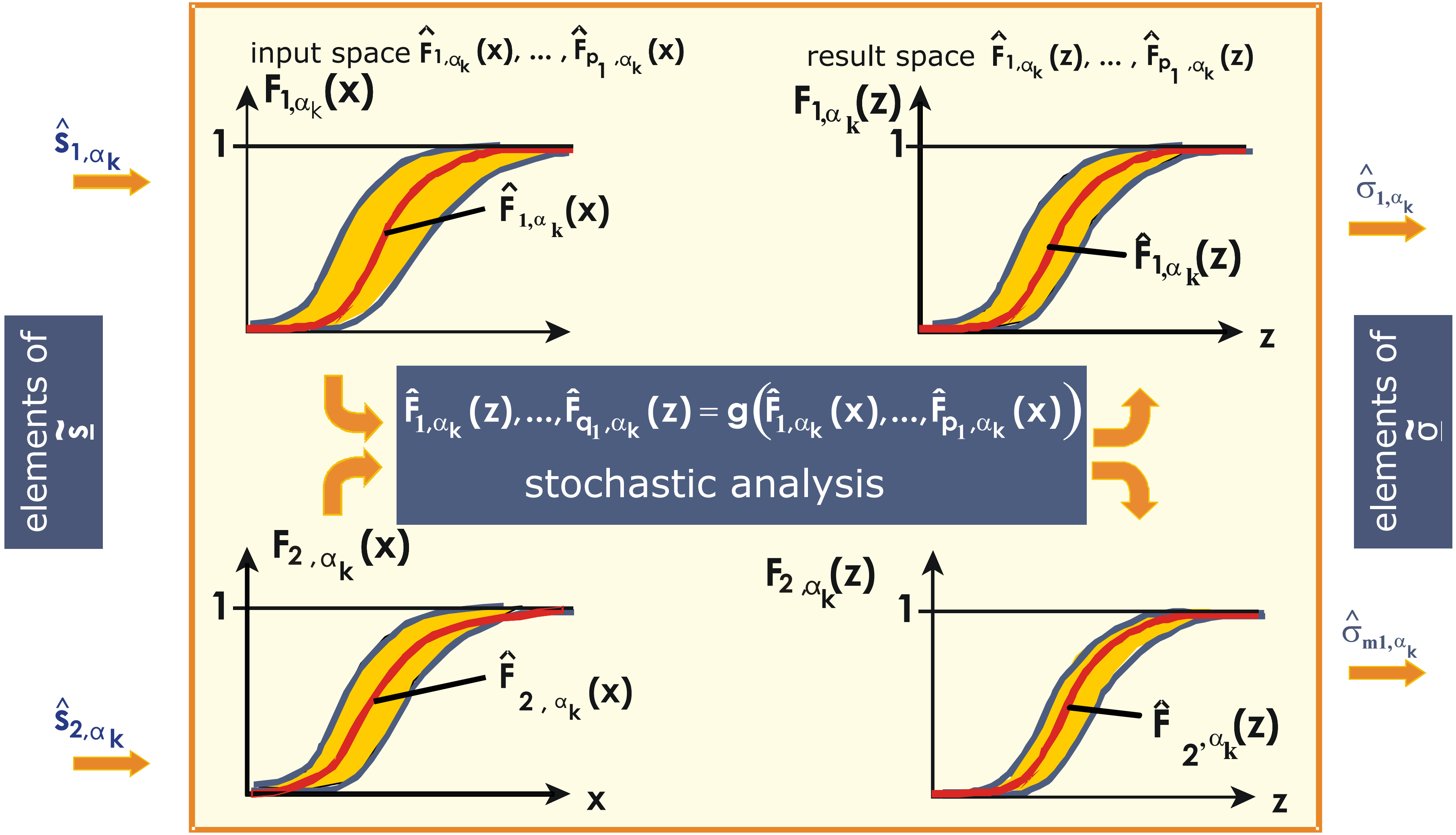
fuzzy random input parameters:

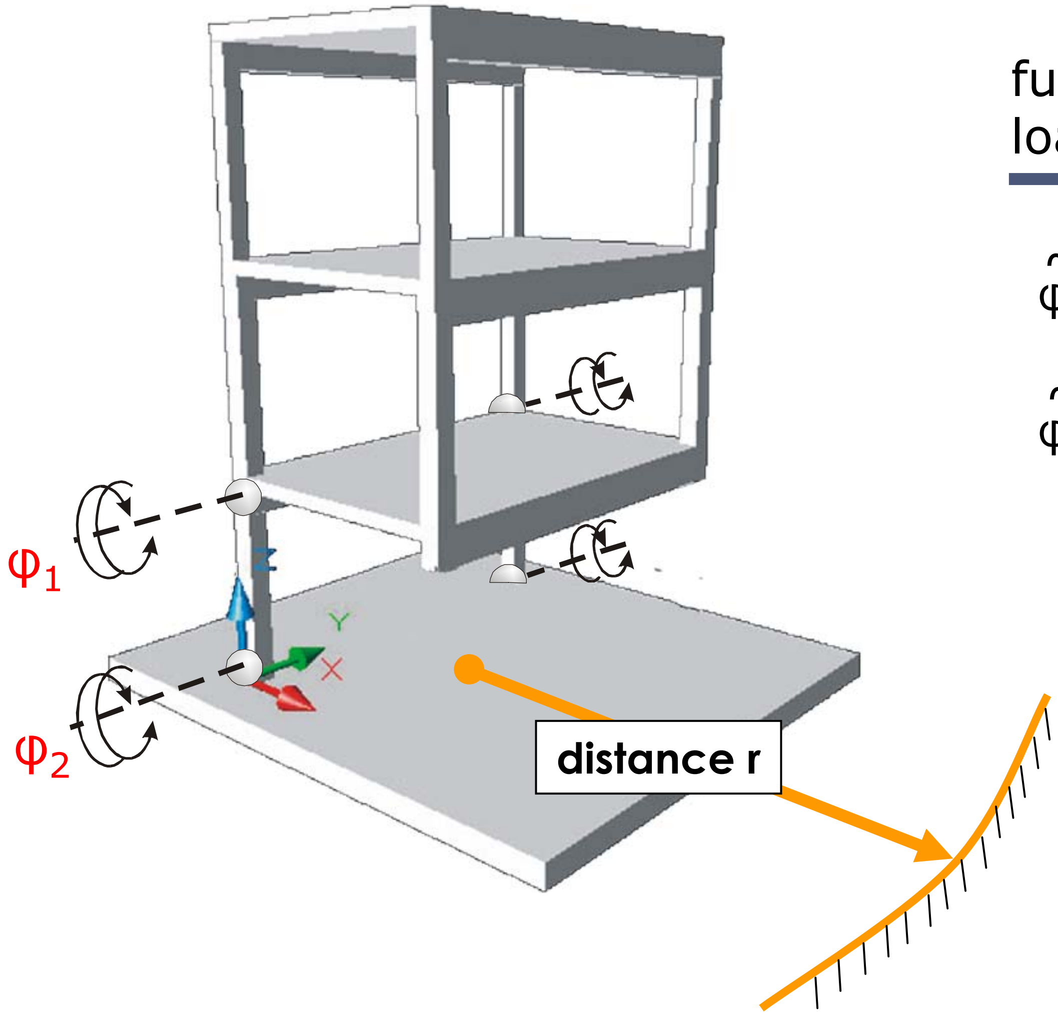
fuzzy probability distribution functions

$$\tilde{F}_1(x) = F_1(x, \tilde{s}_1) \quad \text{and} \quad \tilde{F}_2(x) = F_2(x, \tilde{s}_2)$$



Fuzzy Probabilistic Multibody Dynamics (3)





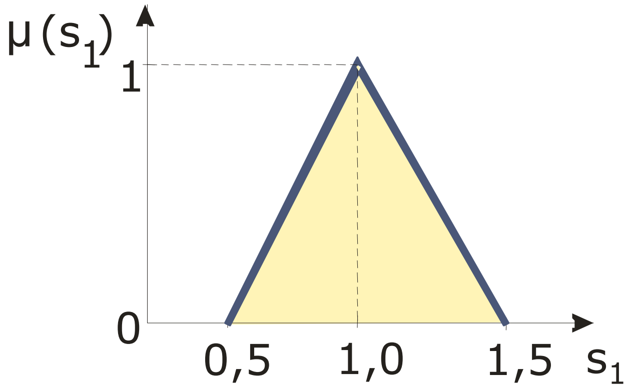
fuzzy load-displacement-dependencies

$$\tilde{\varphi}_1(M) = \tilde{s}_1 \cdot \varphi_1(M)$$

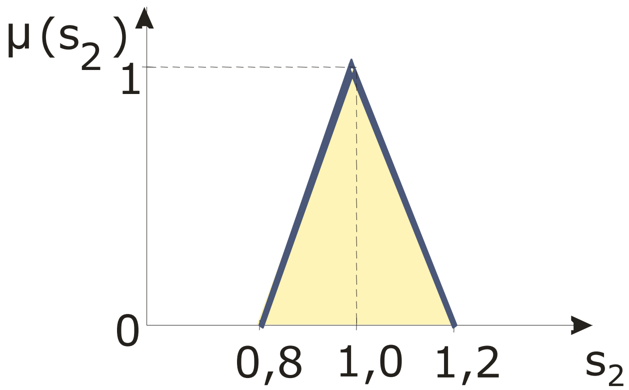
$$\tilde{\varphi}_2(M) = \tilde{s}_2 \cdot \varphi_2(M)$$

Fuzzy Multi Body Dynamics (2) – Example 1

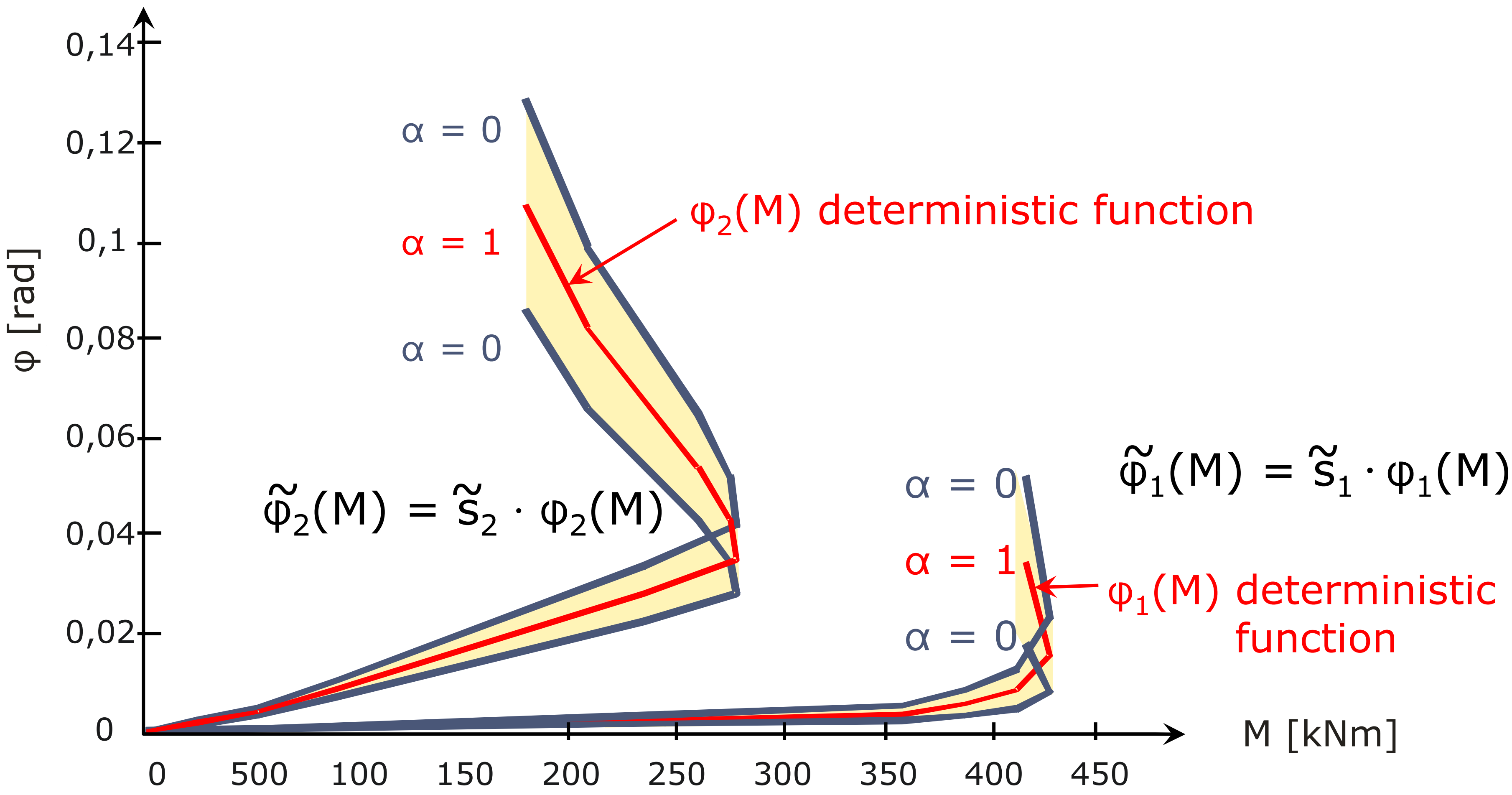
fuzzy bunch parameter $\tilde{s}_1$



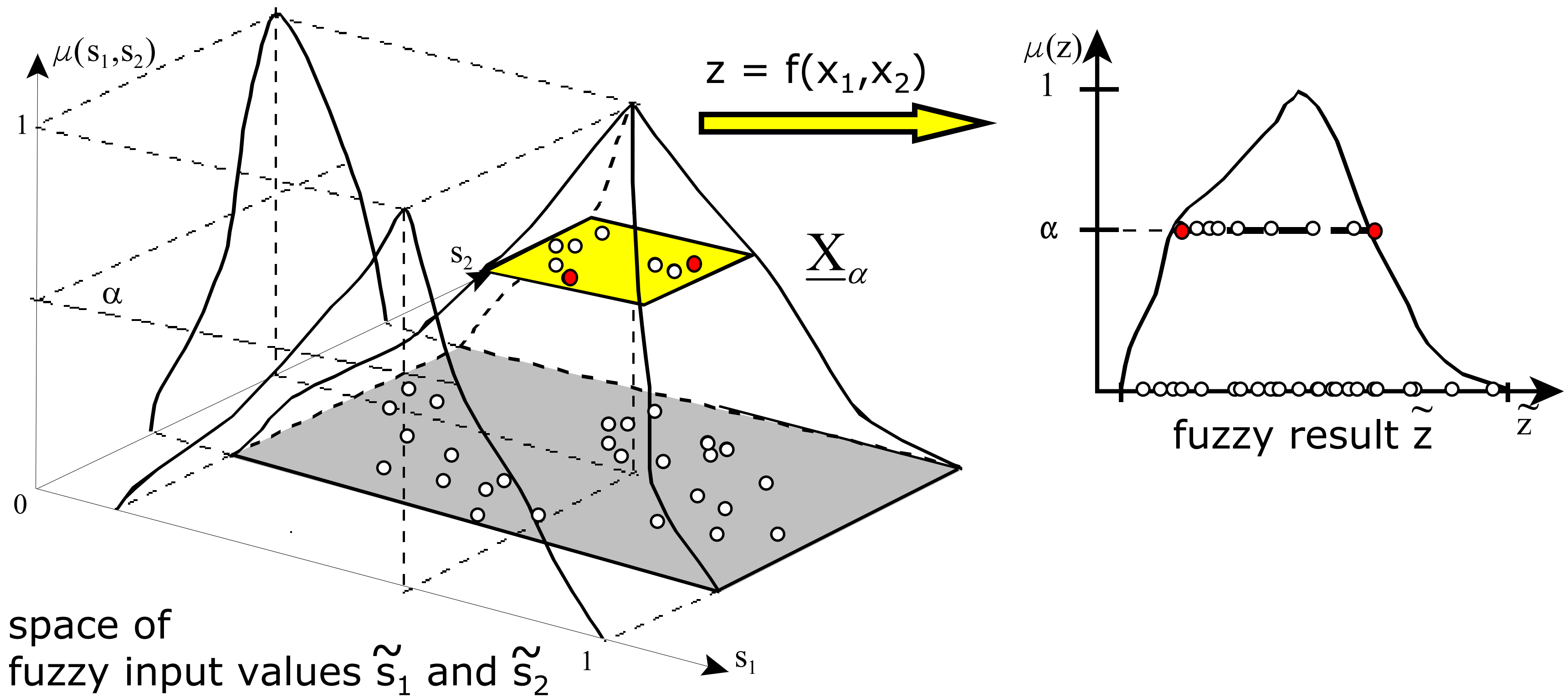
fuzzy bunch parameter $\tilde{s}_2$



fuzzy load displacement relation



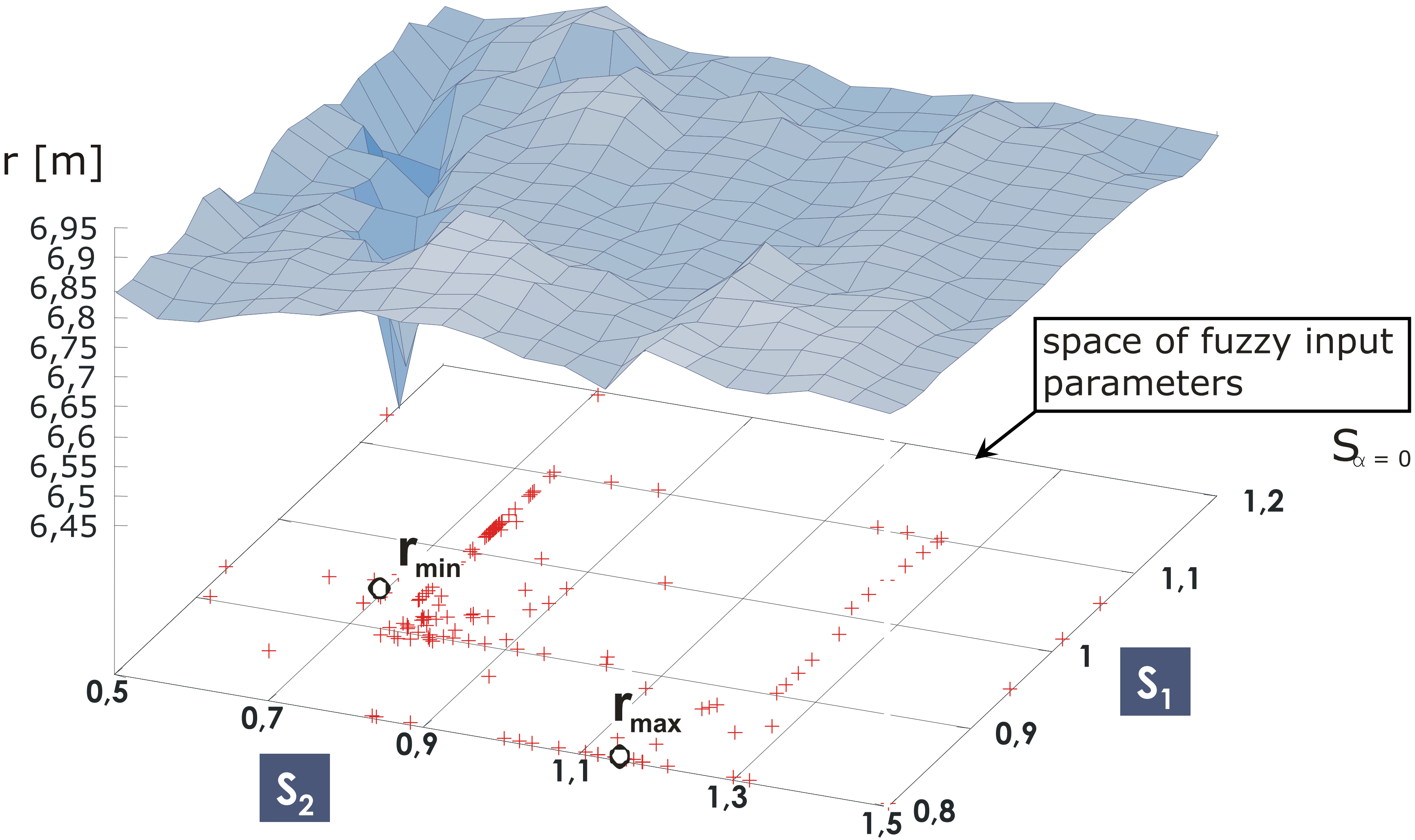
Fuzzy Multi Body Dynamics (2)



- objective function:

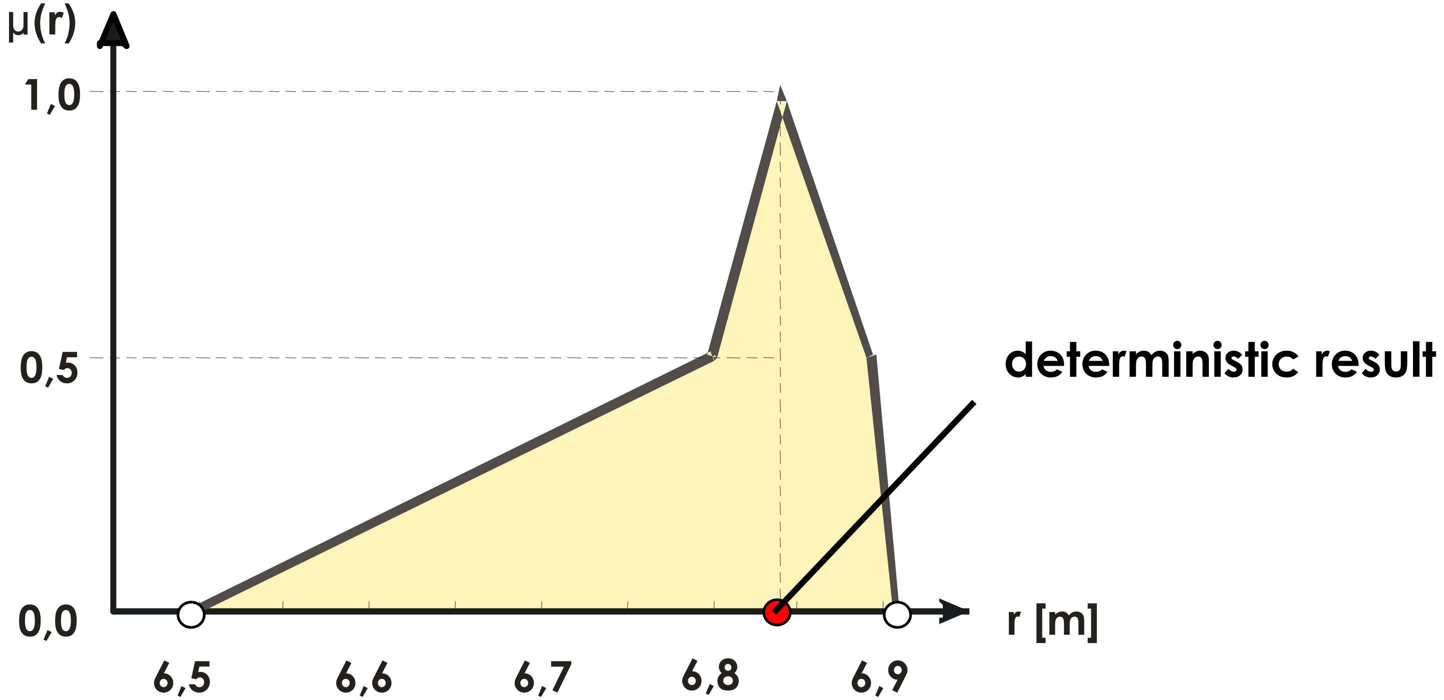
$$z_j = f_j(x_1; \ldots; x_n) \Rightarrow \max \mid (x_1; \ldots; x_n) \in \underline{X}_\alpha$$

$$z_j = f_j(x_1; \ldots; x_n) \Rightarrow \min \mid (x_1; \ldots; x_n) \in \underline{X}_\alpha$$

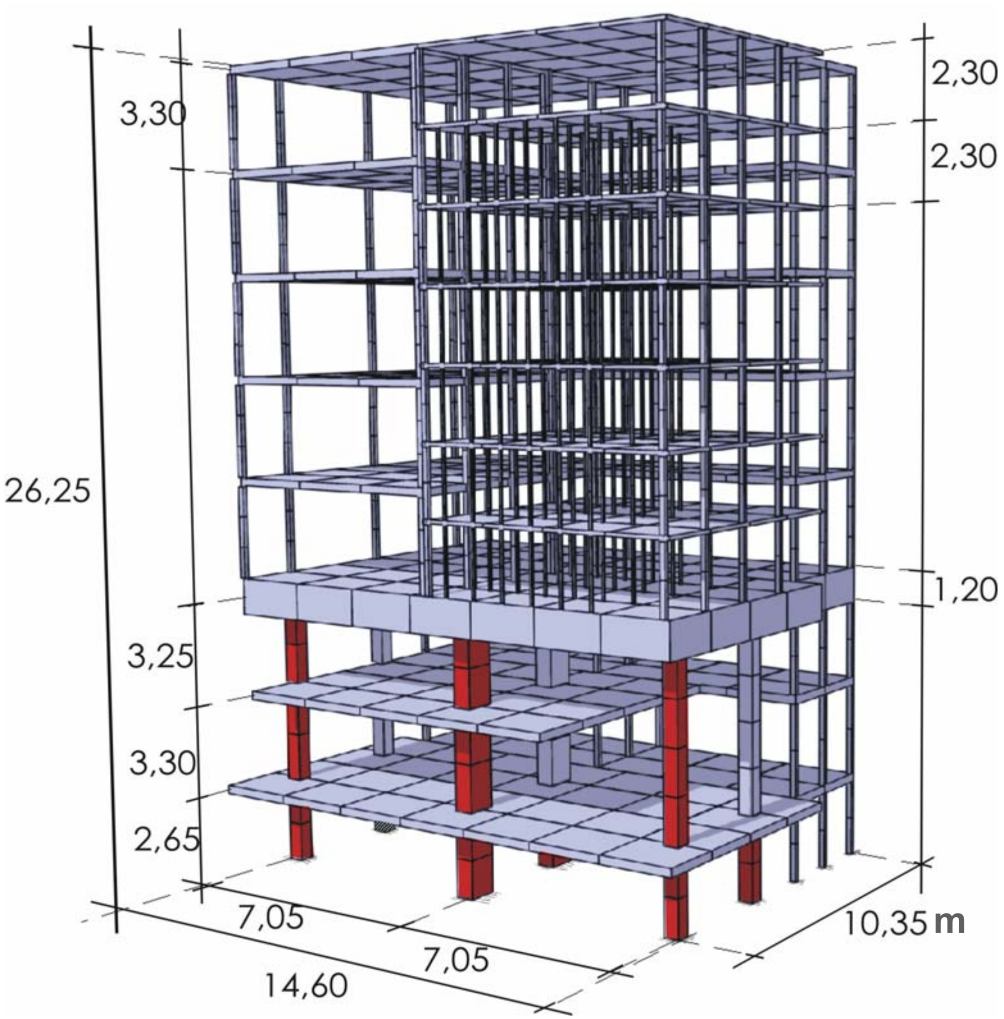


r [m]
6,95
6,9
6,85
6,8
6,75
6,7
6,65
6,6
6,55
6,5
6,45
space of fuzzy input parameters
$S_{\alpha = 0}$
r_{min}
r_{max}
0,5
0,7
0,9
1,1
1,3
1,5
S_2
0,8
0,9
1
1,1
1,2
S_1

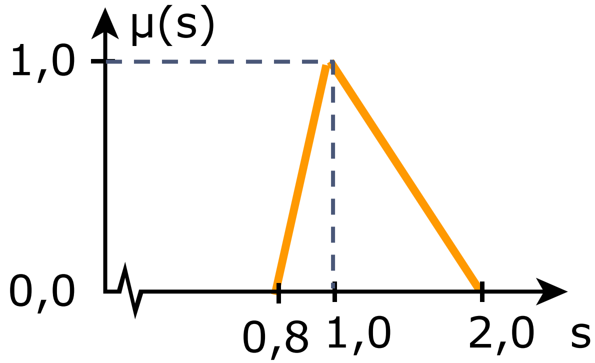
distance r



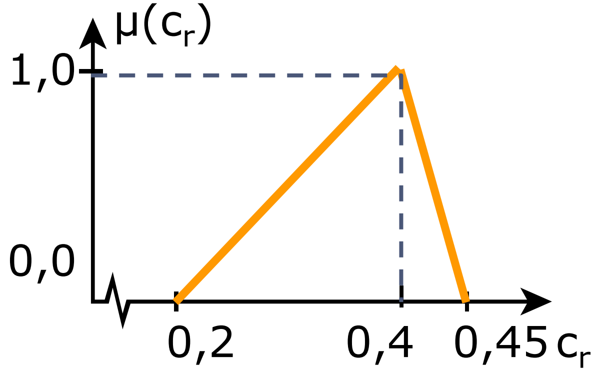
Fuzzy Multibody Dynamics (6) – Example 1



fuzzy bunch parameter of the load-displacement-dependencies

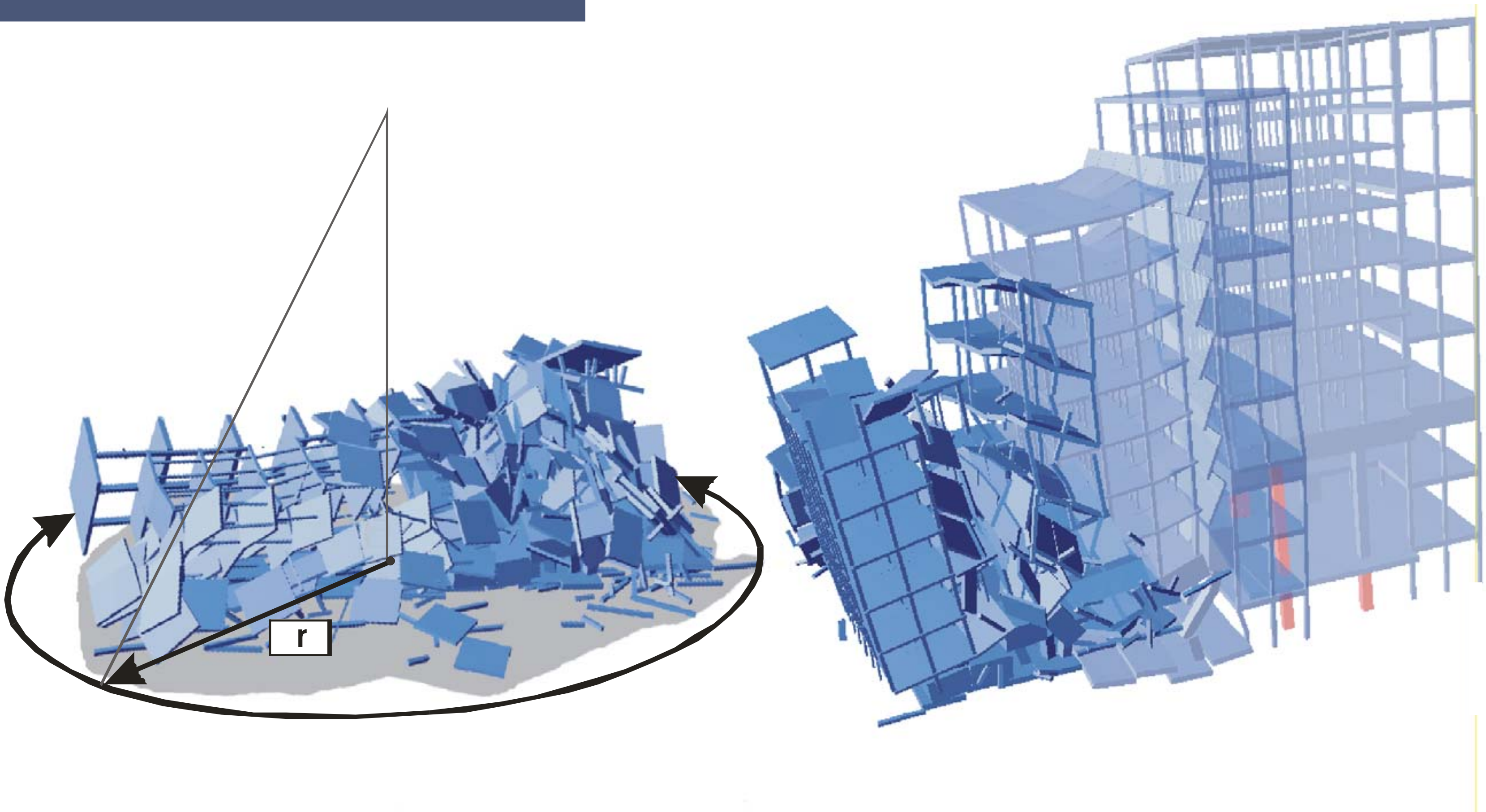


fuzzy friction parameter

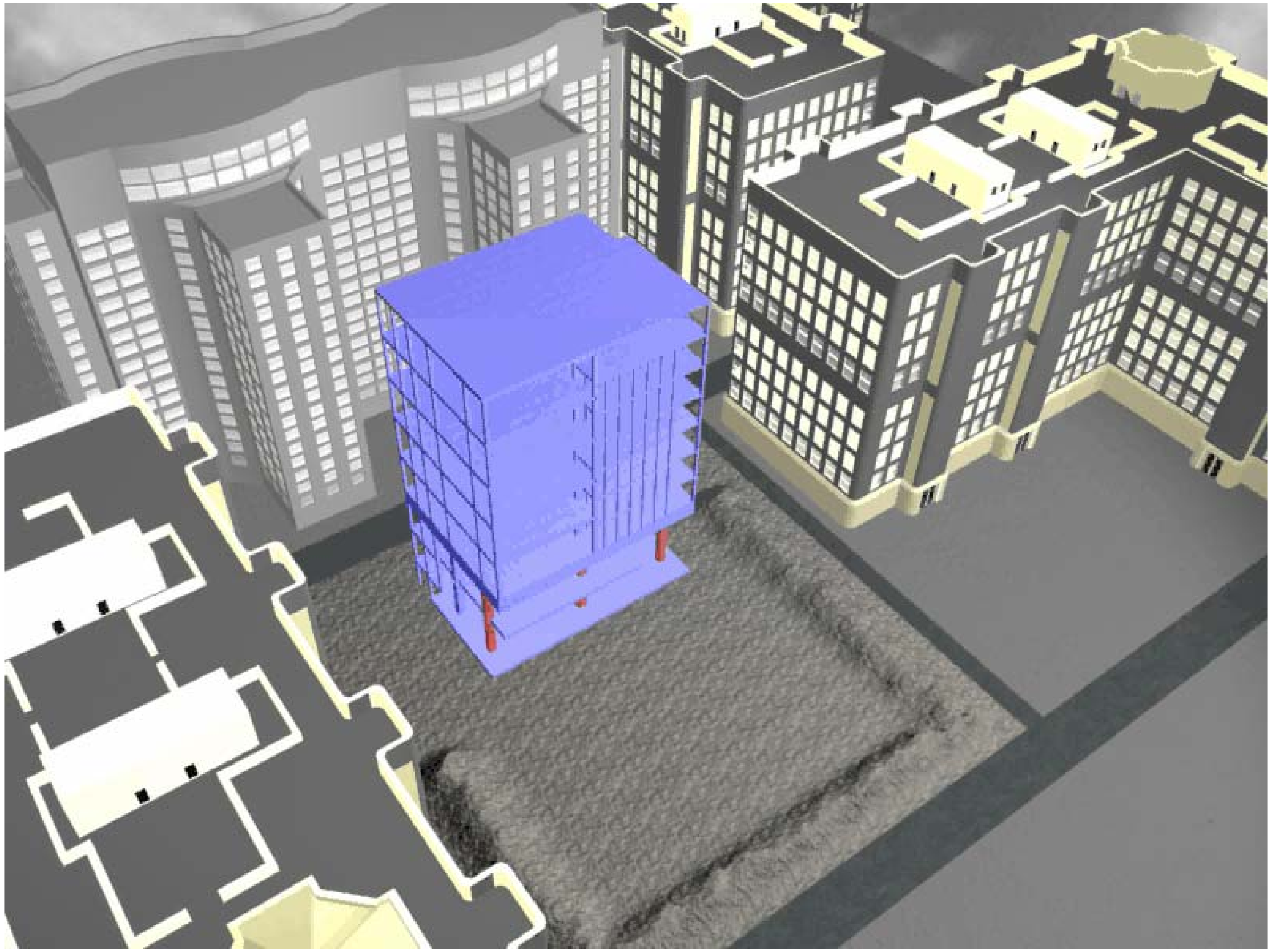


Fuzzy Multibody Dynamics (7) – Example 1

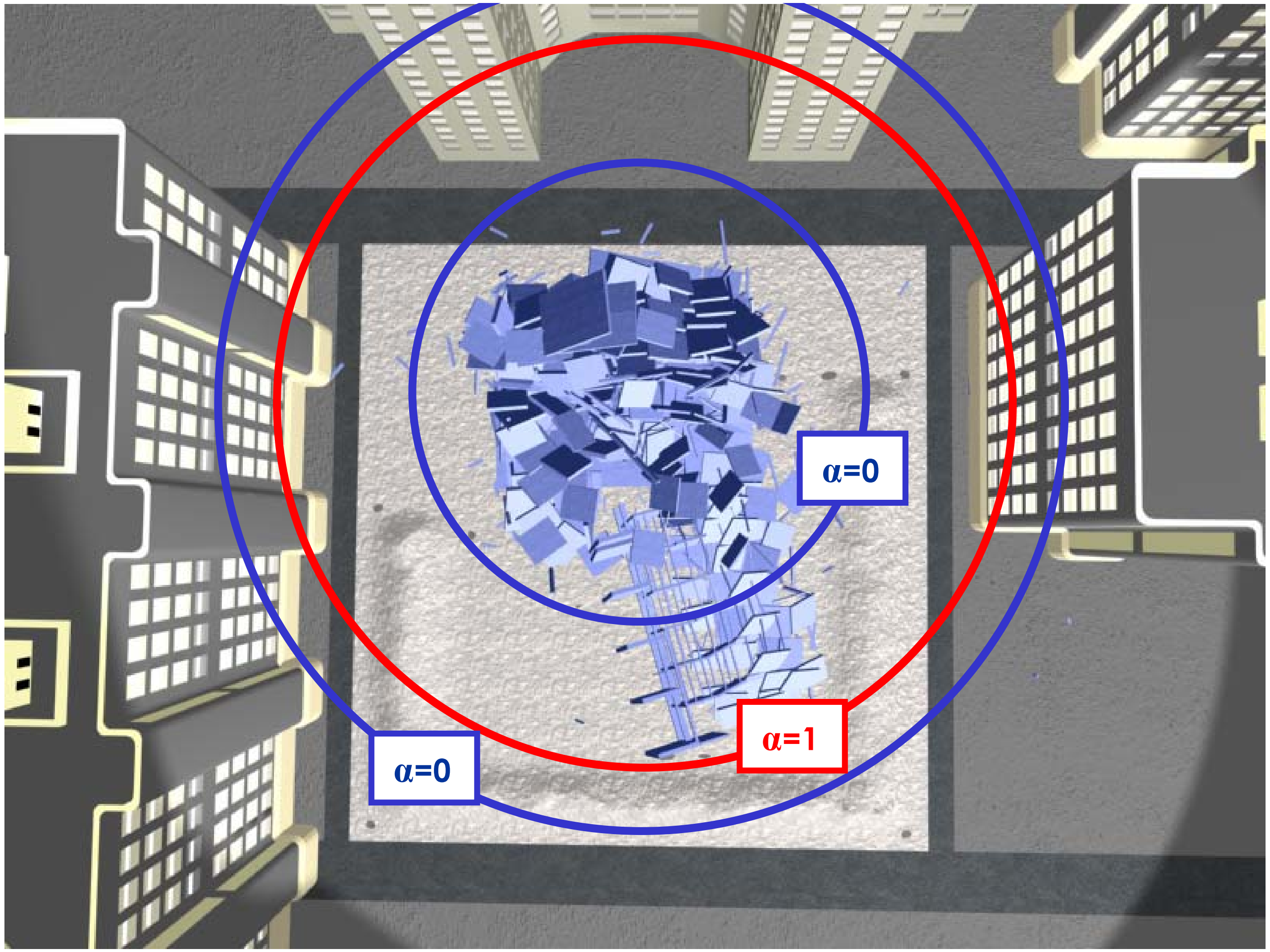
debris distance radius



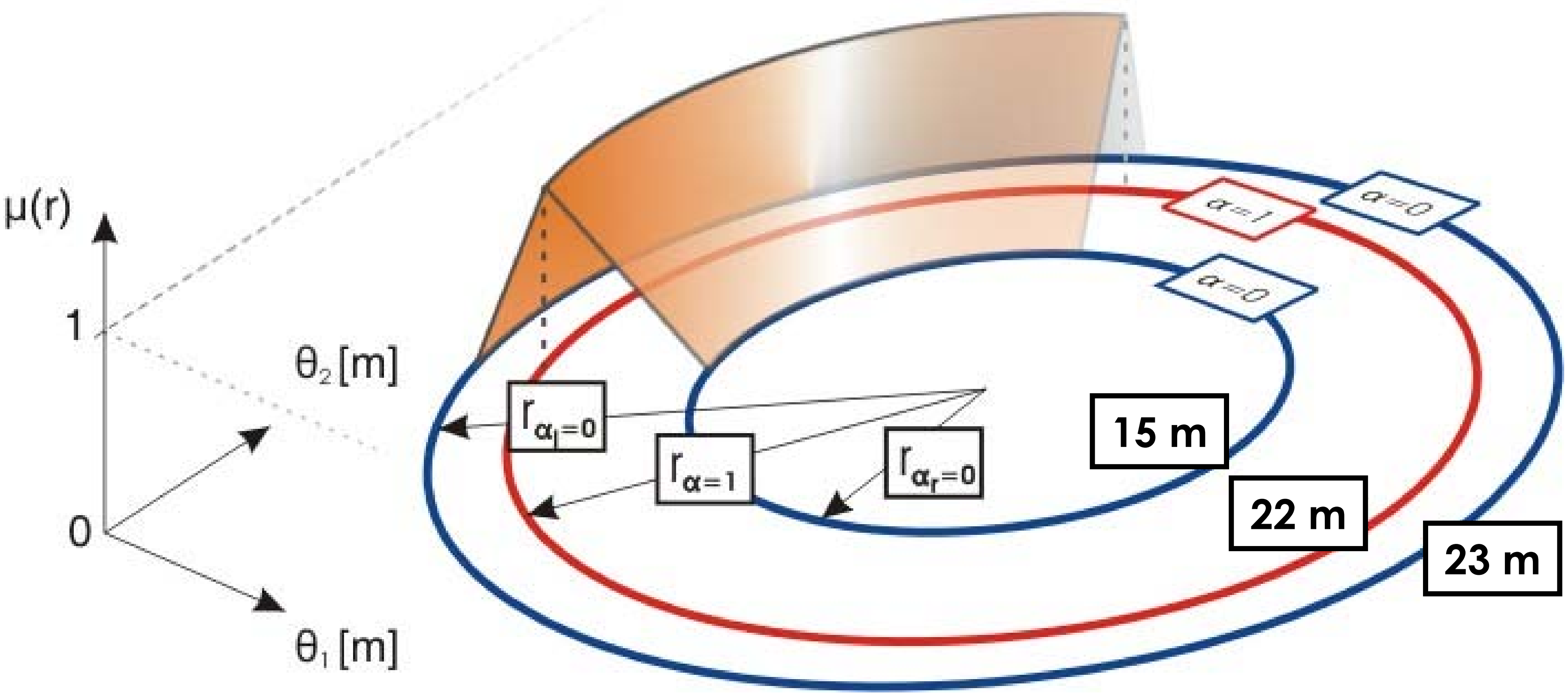
Fuzzy Multibody Dynamics (8) – Example 1

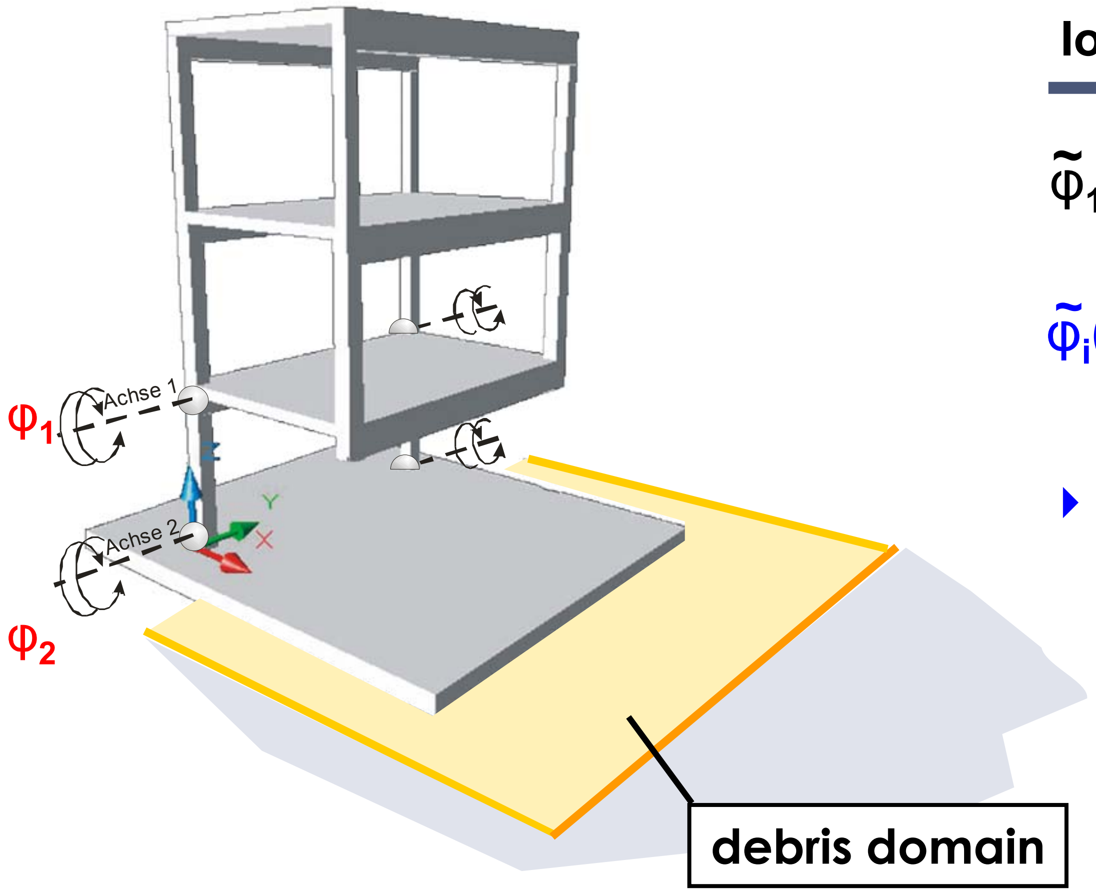


Fuzzy Multibody Dynamics (9) – Example 1



fuzzy distance $\tilde{r}$



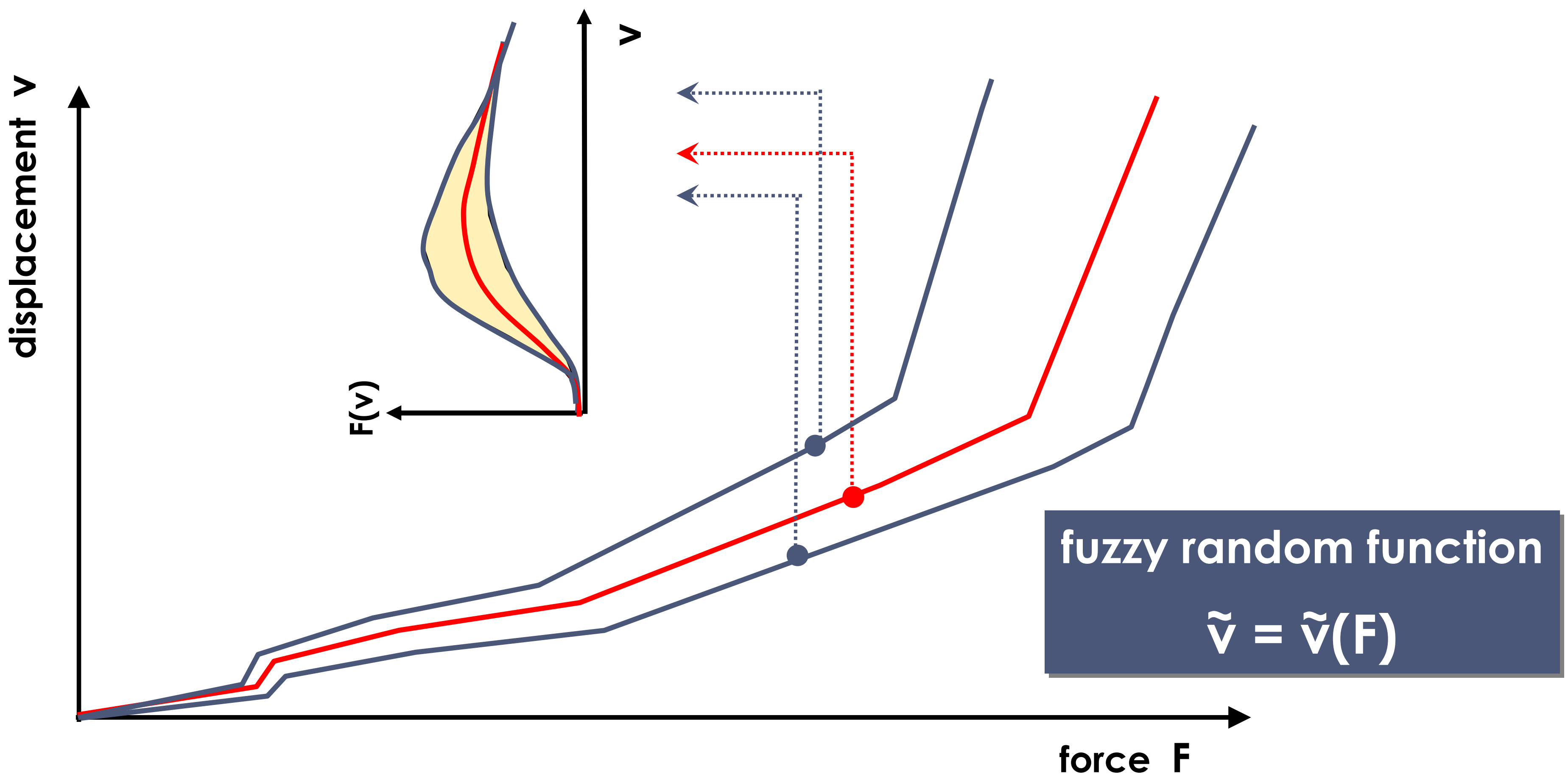


load displacement relation

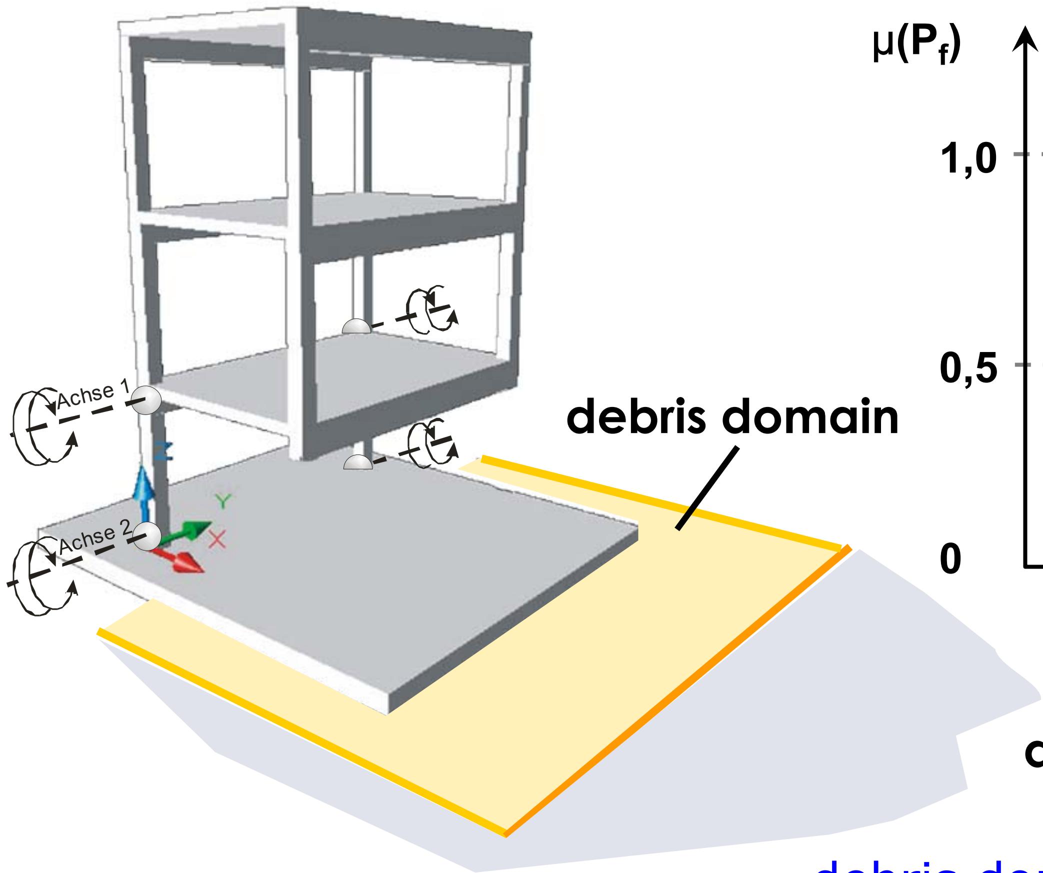
$\tilde{\varphi}_1(M,\tilde{s}_1)$ und $\tilde{\varphi}_2(M,\tilde{s}_2)$

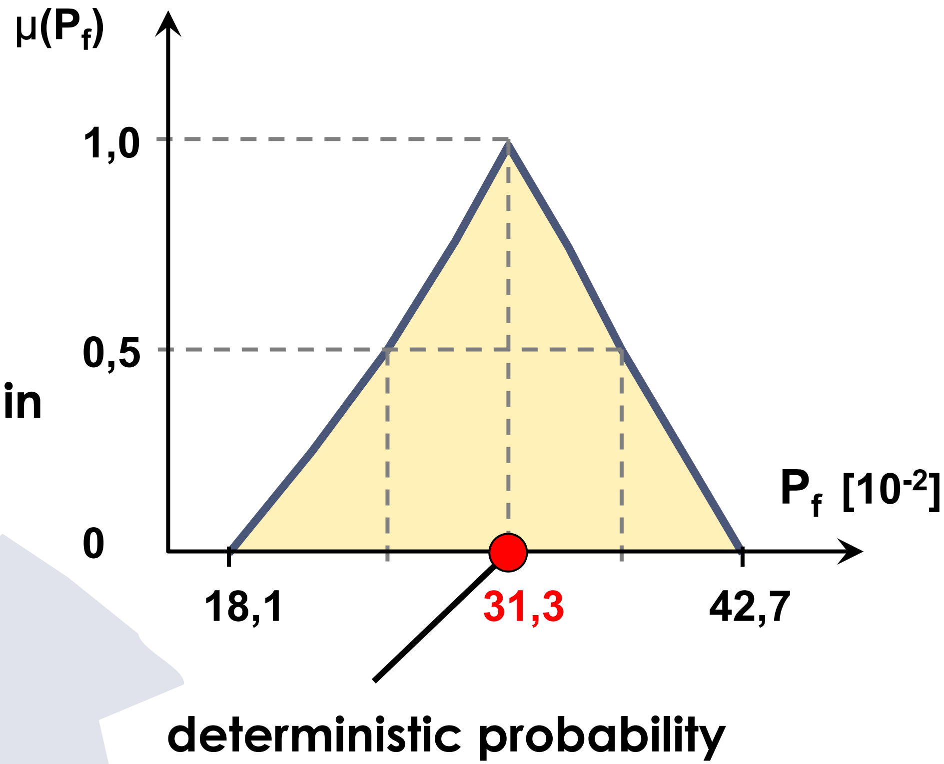
$\tilde{\varphi}_i(M,\tilde{s}_i)$ are modeled as fuzzy random function

- uncertain lognormal distribution with fuzzy standard devations $\tilde{s}$


displacement v
v
F(v)
force F
fuzzy random function
ṽ = ṽ(F)

fuzzy probability, that parts of debris fall outside the debris domain





debris domain is insufficiently designed

Conclusions

- In the case of data uncertainty fuzziness and fuzzy randomness are useful mathematical models

- The application on blasting processes demonstrates the information profit in practicel problems

Thank you !