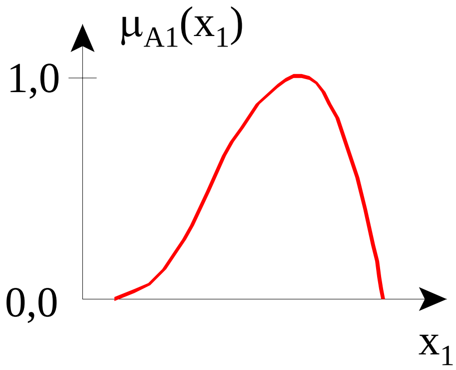
Cartesian product

$$\tilde{K} = \tilde{A}_1 \times \tilde{A}_2 \times \ldots \times \tilde{A}_n$$

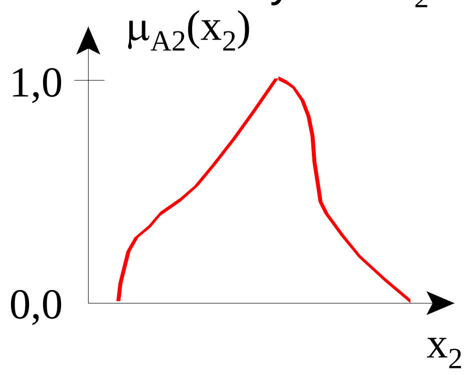
$$\tilde{K} = \left\{ \left[\underline{x} = (x_1; x_2; \ldots; x_n); \mu_K(\underline{x}) = \mu_K(x_1; x_2; \ldots; x_n) \right] \;\middle|\; x_i \in X_i; \mu_K(\underline{x}) = \min\left[\mu_{Ai}(x_i)\right]; i = 1; \ldots; n \right\}$$

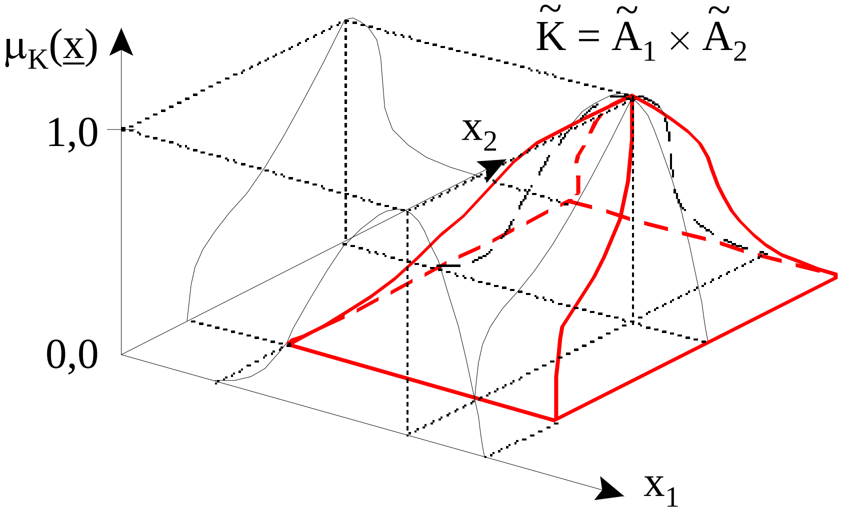
Fuzzy sets

Fuzzy set $\tilde{A}_1$



Fuzzy set $\tilde{A}_2$





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Extension principle

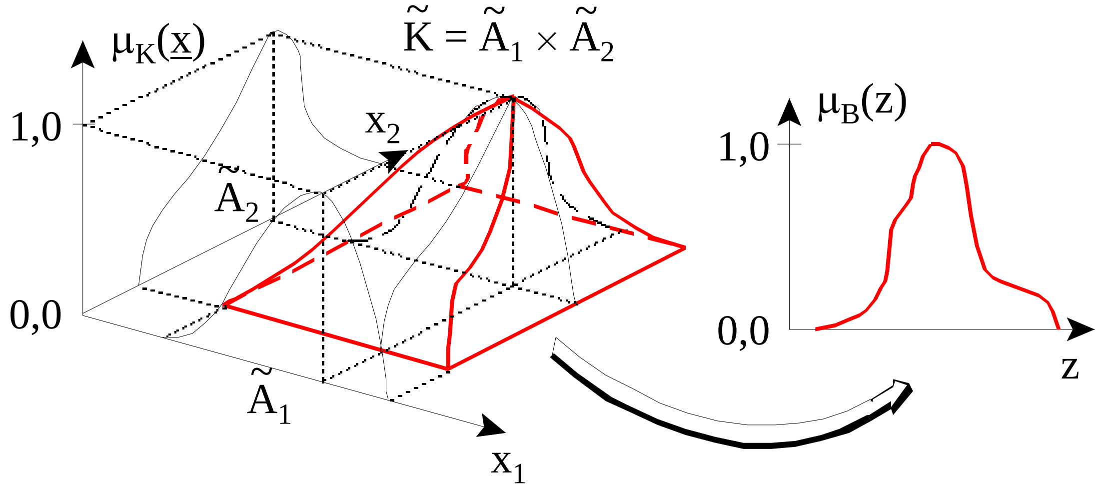
$$\tilde{K} \rightarrow \tilde{B} \;\middle|\; \tilde{K} \subseteq X_1 \times \ldots \times X_n ; \tilde{B} \subseteq Z$$

$$\tilde{B} = \left\{ (z; \mu_B(z)) \;\middle|\; z = f(x_1; \ldots; x_n) ; z \in Z; (x_1; \ldots; x_n) \in X_1 \times \ldots \times X_n \right\}$$

$$\mu_B(z) = \begin{cases} \sup\limits_{z=f(x_1;\ldots;x_n)} \min \, [\mu_1(x_1); \ldots; \mu_n(x_n)], & \text{if } \exists \, z = f(x_1; \ldots; x_n) \\ 0 & \text{otherwise} \end{cases}$$

Fuzzy input sets
Cartesian product

Fuzzy result set



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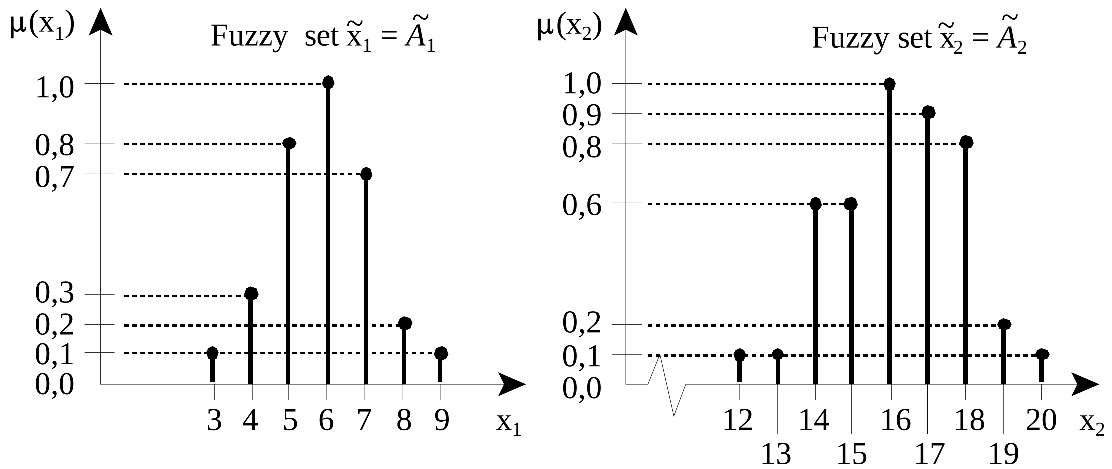
Extension principle - example

Fuzzy input sets (discrete)



$$z = f(x_1; x_2) = 3 \cdot x_1 - x_2 + 5$$

	(3; 0,1)	(4; 0,3)	(5; 0,8)	(6; 1,0)	(7; 0,7)	(8; 0,2)	(9; 0,1)
(12; 0,1)	(2; 0,1)						
(13; 0,1)							
(14; 0,6)							
(15; 0,6)							
(16; 1,0)							
(17; 0,9)							
(18; 0,8)							
(19; 0,2)							
(20; 0,1)							

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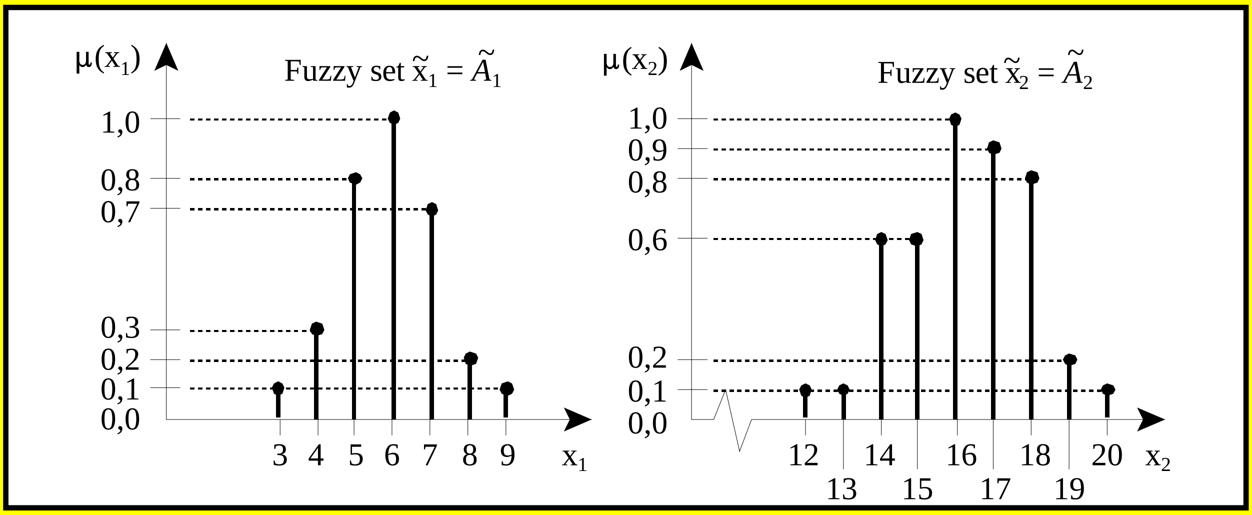
Extension principle - example

Fuzzy input sets



$$\mathbf{z = f(x_1; x_2) = 3 \cdot x_1 - x_2 + 5}$$

	(3; 0,1)	(4; 0,3)	(5; 0,8)	(6; 1,0)	(7; 0,7)	(8; 0,2)	(9; 0,1)
(12; 0,1)	(2; 0,1)	(5; 0,1)	(8; 0,1)	(11; 0,1)	(14; 0,1)	(17; 0,1)	(20; 0,1)
(13; 0,1)	(1; 0,1)	(4; 0,1)	(7; 0,1)	(10; 0,1)	(13; 0,1)	(16; 0,1)	(19; 0,1)
(14; 0,6)	(0; 0,1)	(3; 0,3)	(6; 0,6)	(9; 0,6)	(12; 0,6)	(15; 0,2)	(18; 0,1)
(15; 0,6)	(-1; 0,1)	(2; 0,3)	(5; 0,6)	(8; 0,6)	(11; 0,6)	(14; 0,2)	(17; 0,1)
(16; 1,0)	(-2; 0,1)	(1; 0,3)	(4; 0,8)	(7; 1,0)	(10; 0,7)	(13; 0,2)	(16; 0,1)
(17; 0,9)	(-3; 0,1)	(0; 0,3)	(3; 0,8)	(6; 0,9)	(9; 0,7)	(12; 0,2)	(15; 0,1)
(18; 0,8)	(-4; 0,1)	(-1; 0,3)	(2; 0,8)	(5; 0,8)	(8; 0,7)	(11; 0,2)	(14; 0,1)
(19; 0,2)	(-5; 0,1)	(-2; 0,2)	(1; 0,2)	(4; 0,2)	(7; 0,2)	(10; 0,2)	(13; 0,1)
(20; 0,1)	(-6; 0,1)	(-3; 0,1)	(0; 0,1)	(3; 0,1)	(6; 0,1)	(9; 0,1)	(12; 0,1)

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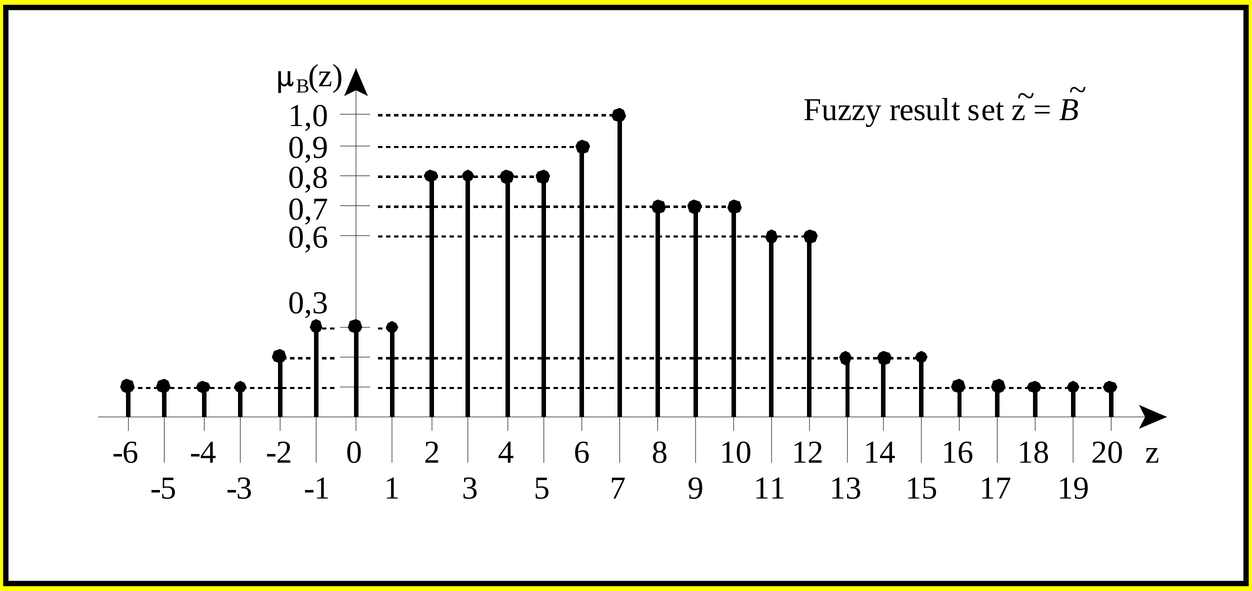
Extension principle - example

Fuzzy result set



	(3; 0,1)	**(4; 0,3)**	**(5; 0,8)**	**(6; 1,0)**	**(7; 0,7)**	**(8; 0,2)**	**(9; 0,1)**
(12; 0,1)	**(2; 0,1)**	**(5; 0,1)**	**(8; 0,1)**	**(11; 0,1)**	**(14; 0,1)**	**(17; 0,1)**	**(20; 0,1)**
(13; 0,1)	**(1; 0,1)**	**(4; 0,1)**	**(7; 0,1)**	**(10; 0,1)**	**(13; 0,1)**	**(16; 0,1)**	**(19; 0,1)**
(14; 0,6)	**(0; 0,1)**	**(3; 0,3)**	**(6; 0,6)**	**(9; 0,6)**	**(12; 0,6)**	**(15; 0,2)**	**(18; 0,1)**
(15; 0,6)	**(-1; 0,1)**	**(2; 0,3)**	**(5; 0,6)**	**(8; 0,6)**	**(11; 0,6)**	**(14; 0,2)**	**(17; 0,1)**
(16; 1,0)	**(-2; 0,1)**	**(1; 0,3)**	**(4; 0,8)**	**(7; 1,0)**	**(10; 0,7)**	**(13; 0,2)**	**(16; 0,1)**
(17; 0,9)	**(-3; 0,1)**	**(0; 0,3)**	**(3; 0,8)**	**(6; 0,9)**	**(9; 0,7)**	**(12; 0,2)**	**(15; 0,1)**
(18; 0,8)	**(-4; 0,1)**	**(-1; 0,3)**	**(2; 0,8)**	**(5; 0,8)**	**(8; 0,7)**	**(11; 0,2)**	**(14; 0,1)**
(19; 0,2)	**(-5; 0,1)**	**(-2; 0,2)**	**(1; 0,2)**	**(4; 0,2)**	**(7; 0,2)**	**(10; 0,2)**	**(13; 0,1)**
(20; 0,1)	**(-6; 0,1)**	**(-3; 0,1)**	**(0; 0,1)**	**(3; 0,1)**	**(6; 0,1)**	**(9; 0,1)**	**(12; 0,1)**

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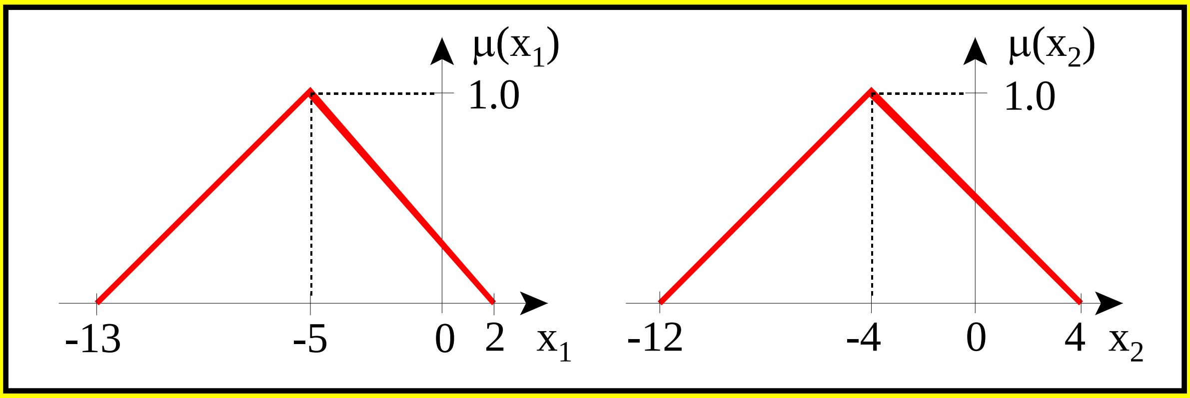
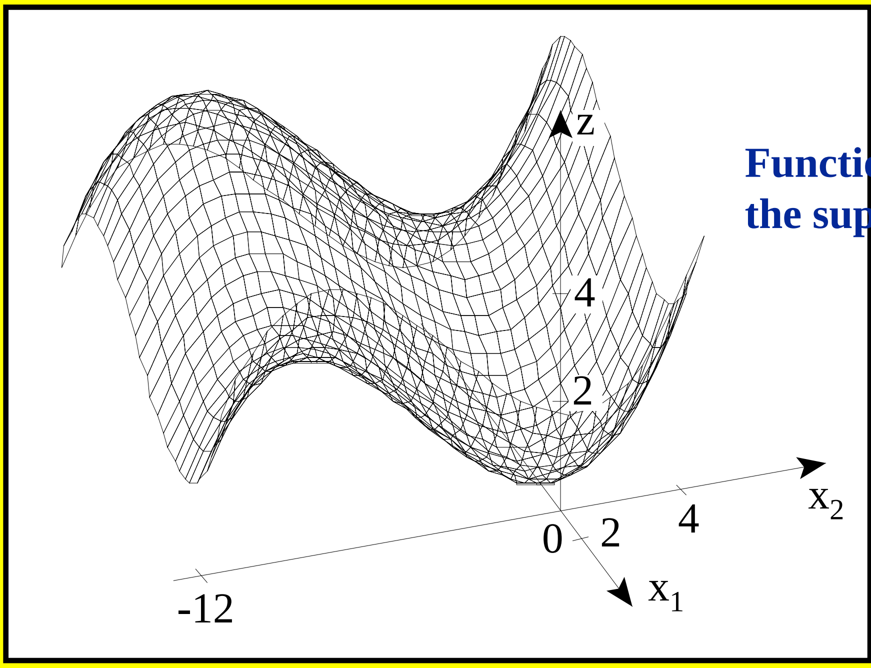
Extension principle - example

fuzzy input values $\tilde{x}_1$ und $\tilde{x}_2$ (continuous)



mapping operator

$$z = f(x_1; x_2) = x_1^3 + 17\,x_1^2 + 48\,x_1 + x_2^3 + 13\,x_2^2 + 3\,x_2 + 65$$



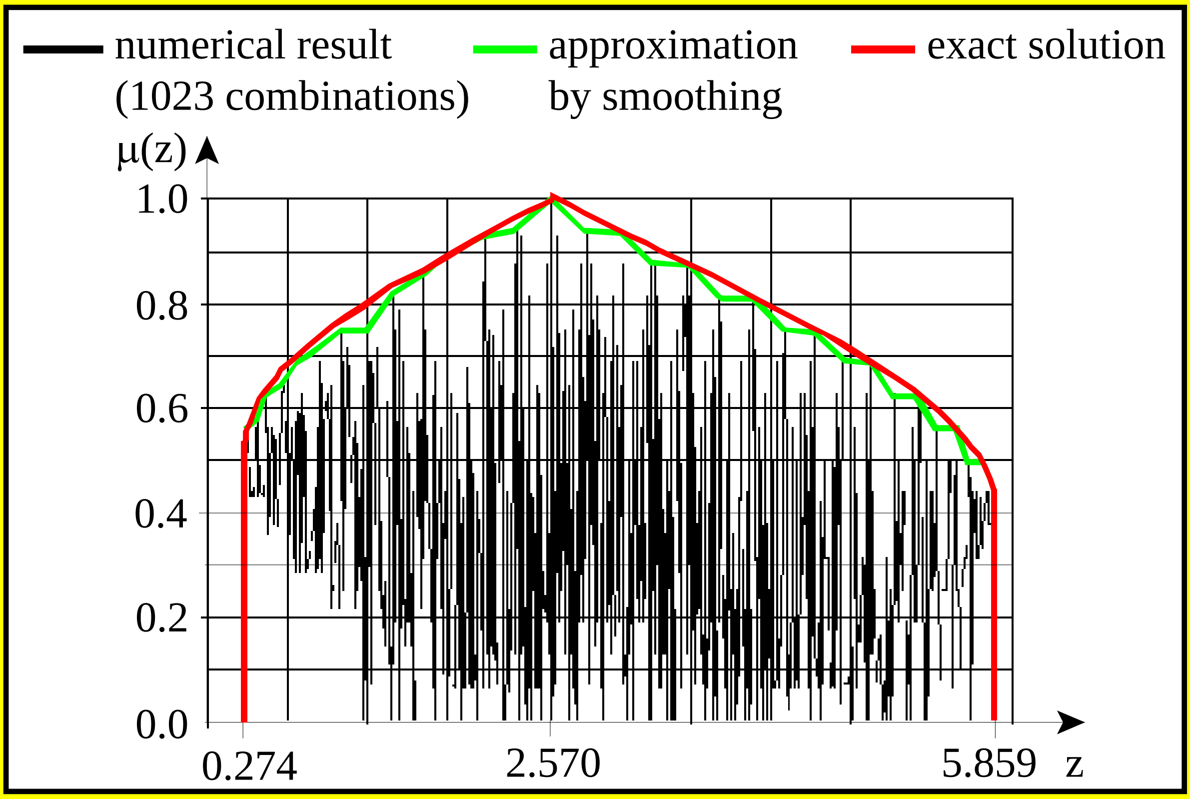
Function about the support

Extension principle - example

numerical procedure to compute the fuzzy result

- discretization of the support of all fuzzy input values
- generation of all combinations of discretized elements
- determination of the membership values using the min operator
- computation of the results from all element combinations using the mapping operator
- determination of the membership values of the result elements by applying the max operator
- generation of the membership function for the fuzzy result

fuzzy result value $\tilde{z}$



problems: tremendous high numerical effort
exactness of the result

Dresden University of Technology
Institute of Structural Analysis