How to Estimate, Take Into Account, and Improve Travel Time Reliability in Transportation Networks

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Decreasing Traffic . . . Traffic Assignment: . . . Towards a More . . . A Seemingly Natural . . . A More Realistic . . . Taking Uncertainty . . . Logit Discrete Choice . . . Towards an Optimal . . . Exponential Disutility . . .



- 1. Decreasing Traffic Congestion: Formulation of the Problem
 - Practical problem: decreasing traffic congestion.
 - *Important difficulty:* a new road can worsen traffic congestion.
 - *Conclusion:* importance of the preliminary analysis of the results of road expansion.
 - *Traditional approach* assumes that we know:
 - the exact amount of traffic going from zone A to zone B (*OD-matrix*), and
 - the exact capacity of each road segment.
 - *Limitations:* in practice, we only know all this with uncertainty.
 - What we do: we show how to take this uncertainty into account in traffic simulations.



2. Traffic Assignment: Brief Reminder

- Traffic demand: # of drivers d_{ij} who need to go from zone *i* to zone *j* origin-to-destination (O-D) matrix.
- *Capacity* of a road link the number *c* of cars per hour which can pass through this link.

• Travel time along a link:
$$t = t^f \cdot \left[1 + a \cdot \left(\frac{v}{c}\right)^{\beta}\right]$$
, where:

•
$$t^f = L/s$$
 is a *free-flow* time (s is the speed limit),

- $a \approx 0.15$ and $\beta \approx 4$ are empirical constants.
- Equilibrium: when
 - the travel time along all used alternative routes is exactly the same, and
 - the travel times along other un-used routes is higher.
- *Algorithms:* there exist efficient algorithms for finding the equilibrium.



- 3. How We Can Use the Existing Traffic Assignment Algorithms to Solve Our Problem: Analysis
 - *Main objective:* predict how different road project affect future traffic congestion.
 - Future traffic demands what is known: there exist techniques for predicting daily O-D matrices.
 - What is lacking: we need to "decompose" the daily O-D matrix into 1 hour (or 15 minute) intervals.
 - 1st approximation: assume that the proportion of drivers starting at, say 6 to 7 am is the same as now.
 - Need for a more accurate approximation:
 - drivers may start early because of congestion;
 - if a new road is built, they will start later;
 - the % of those who start 6–7 am will decrease.
 - We cover: both approximations.

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4. Towards a More Accurate Approximation to O-D Matrices

• Describing preferences: empirical utility formula $u_i = -0.1051 \cdot E(T) - 0.0931 \cdot E(SDE) - 0.1299 \cdot E(SDL) - 1.3466 \cdot P_L - 0.3463 \cdot \frac{S}{E(T)},$

where E(X) means expected value,

- T is the travel time T,
- SDE is the wait time when arriving early,
- $\bullet~SDL$ is the delay when arriving late,
- P_L is the probability of arriving late, and
- S is the variance of the travel time.
- Logit model: the probability P_i that a driver will choose the *i*-th time interval is proportional to $\exp(u_i)$:

$$P_i = \frac{\exp(u_i)}{\exp(u_1) + \ldots + \exp(u_n)}$$



5. A Seemingly Natural Idea and Its Limitations

- Seemingly natural idea:
 - start with the 1st approximation O-D matrices M_1 ;
 - based on M_1 , we find travel times, and use them to find the new O-D matrices $M_2 \stackrel{\text{def}}{=} F(M_1)$;
 - based on M_2 , we find travel times, and use them to find the new O-D matrices $M_3 \stackrel{\text{def}}{=} F(M_2)$;
 - repeat until converges.
- Toy example illustrating a problem:
 - now: no congestion, all start at 7:30, work at 8 am;
 - M_1 : full O-D matrix for 7:30 am, 0 for 7:15 am;
 - based on this M_1 , we get huge delays;
 - M_2 : everyone leaves for work early at 7:15 am;
 - at 7:30, roads are freer, so in M_3 , all start at 7:30;
 - no convergence: $M_1 = M_3 = \ldots \neq M_2 = M_4 \ldots$

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6. A More Realistic Approach

- *Above idea:* drivers make decisions based only on *pre-vious* day traffic.
- *More accurate idea:* drivers make decisions based on the *average* traffic over a few past days.
- Resulting process:
 - start with the 1st approximation O-D matrices M_1 ;

- for
$$i = 2, 3, ...$$
:

* compute the average $E_i = \frac{M_1 + \ldots + M_i}{i}$,

* find traffic times based on E_i ;

- * use these traffic times to compute a new O-D matrix $M_{i+1} = F(E_i);$
- * repeat until converges.
- *Process converges:* on toy examples, on El Paso network, etc.

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7. Algorithm Simplified

• Main idea: once we know the previous average E_i , we can compute

$$E_{i+1} = \frac{(M_1 + \ldots + M_i) + M_{i+1}}{i+1} = \frac{i \cdot E_i + M_{i+1}}{i+1} = E_i \cdot \left(1 - \frac{1}{i+1}\right) + M_{i+1} \cdot \frac{1}{i+1}.$$

• We know: that $M_{i+1} = F(E_i)$.

- Resulting algorithm:
 - start with the 1st approximation O-D matrices

$$E_1 = M_1;$$

- compute
$$E_{i+1} = E_i \cdot \left(1 - \frac{1}{i+1}\right) + F(E_i) \cdot \frac{1}{i+1};$$

- repeat until converges.

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8. Taking Uncertainty into Account

• Deterministic model:
$$t = t^f \cdot \left[1 + a \cdot \left(\frac{v}{c}\right)^{\beta}\right]$$
.

- Traffic assignment: a driver minimizes the travel time $t = t_1 + \ldots + t_n$.
- In practice: travel times vary.
- Decision theory: maximize expected utility E[u].
- How utility depends on travel time: u(t) = -U(t), where $U(t) = \exp(\alpha \cdot t)$.
- Conclusion: the driver minimizes $E[U(t)] = E[\exp(\alpha \cdot t)] = E[\exp(\alpha \cdot (t_1 + \ldots + t_n)] = E[\exp(\alpha \cdot t_1) \cdot \ldots \cdot \exp(\alpha \cdot t_n)].$
- Deviations on different links are independent, so $E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)].$

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9. Taking Uncertainty into Account (cont-d)

• Minimizing $E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)]$ \Leftrightarrow minimizing $\sum_{i=1}^{n} \tilde{t}_i$, where $\tilde{t}_i \stackrel{\text{def}}{=} \ln(E[\exp(\alpha \cdot t_i)])$.

•
$$\widetilde{t}$$
 depends on t^f and $r \stackrel{\text{def}}{=} \frac{\overline{t} - t^f}{t}$: $\widetilde{t} = F(t^f, r)$.

- If we divide a link into sublinks, we conclude that $F(t_1^f + t_2^f, r) = F(t_1^f, r) + F(t_2^f, r)$, hence $\tilde{t} = t^f \cdot k(r)$.
- For no-congestion case r = 0, we have $\tilde{t} = t^f$, so k(0) = 1 and $k(r) = 1 + a_0 \cdot r + a_2 \cdot r_2 + \dots$
- Empirical analysis: $a_1 \approx 1.4, b \approx 0$, so

$$\widetilde{t} = t^f \cdot \left[1 + a \cdot a_1 \cdot \left(\frac{v}{c}\right)^{\beta} \right]$$

• Solution: use the standard travel time formula with $a \cdot a_1 \approx 0.21$ instead of $a \approx 0.14$.

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10. Acknowledgments

This work was supported in part by:

- by Texas Department of Transportation contract No. 0-5453,
- by National Science Foundation grants HRD-0734825, EAR-0225670, and EIA-0080940,
- by the Japan Advanced Institute of Science and Technology (JAIST) International Joint Research Grant 2006-08, and
- and by the Max Planck Institut für Mathematik.

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11. Logit Discrete Choice Model: A New Justification

- *Reasonable assumption:* if we add same incentive to all routes, probabilities will not change.
- For 2 routes: $P_1 = F(\Delta V)$, where $\Delta V \stackrel{\text{def}}{=} V_1 V_2$.
- Bayes theorem:

$$P(H_i | E) = \frac{P(E | H_i) \cdot P_0(H_i)}{P(E | H_1) \cdot P_0(H_1) + \ldots + P(E | H_n) \cdot P_0(H_n)}$$

• *Idea:* if we add an incentive v_0 to one of the routes, this changes the probability of selecting this route:

$$F(\Delta V + v_0) = \frac{A(v_0) \cdot F(\Delta V)}{A(v_0) \cdot F(\Delta V) + B(v_0) \cdot (1 - F(\Delta V))}.$$

• Conclusion: $F(\Delta V) = \frac{1}{1 + e^{-\beta \cdot \Delta V}}$, so

$$p_1 = F(V_1 - V_2) = \frac{e^{\beta \cdot V_1}}{e^{\beta \cdot V_1} + e^{\beta \cdot V_2}}$$

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12. Towards an Optimal Algorithm for Computing Fixed Points

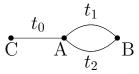
- *Idea:* when iterations $x_{k+1} = f(x_k)$ do not converge, $x_{k+1} = x_k + \alpha \cdot (f(x_k) - x_k) = (1 - \alpha_k) \cdot x_k + \alpha_k \cdot f(x_k).$
- Question: which choice of α_k is best?
- *Idea:* this is a discrete approximation to a continuoustime system $\frac{dx}{dt} = \alpha(t) \cdot (f(x) - x).$
- Scale invariance: the system should not change if we use a different discretization, i.e., re-scale t to $t' = t/\lambda$:

$$\frac{dx}{dt'} = (\lambda \cdot \alpha(\lambda \cdot t')) \cdot (f(x) - x).$$

- Conclusion: $\lambda \cdot \alpha(\lambda \cdot t') = a(t')$, so for $\lambda = 1/t'$, we get $\alpha(t') = \frac{c}{t'}$ for some c.
- Fact: this is exactly what we used: $\alpha_k = 1/k$.

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- 13. Exponential Disutility Functions in Transportation Modeling: Justification
 - Situation:



• *Reasonable assumption:* the driver starting at C will choose the same road as the driver starting at A.

• Formally: if
$$E[u(t_1)] < E[u(t_2)]$$
 then
 $E[u(t_1 + t_0)] < E[u(t_2 + t_0)].$

• Result:
$$u(t) = t$$
, $u(t) = \exp(c \cdot t)$, or
$$u(t) = -\exp(-c \cdot t).$$

• *Fact:* this is exactly the empirically justified formula used in transportation.

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